Advanced Wood-Based Solutions for Mid-Rise and High-Rise Construction: Modelling of Timber Connections under Force and Fire

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SUMMARY

FPInnovations carried out a survey with consultants and researchers on the use of analytical models and software packages related to the analysis and design of mass timber buildings. The responses confirmed that a lack of suitable models and related information for material properties of timber connections, in particular under combination of various types of loads and fire, was creating an impediment to the design and construction of this type of buildings. Furthermore, there is currently a lack of computer models for use in performance-based design for wood buildings, in particular, seismic and fire performance-based design.

In this study, a sophisticated constitutive model for wood-based composite material under stress and temperature was developed. This constitutive model was programmed into a user-subroutine and can be added to most general-purpose finite element software. The developed model was used to model the structural performance of a laminated veneer lumber (LVL) beam and a glulam bolted connection under force and/or fire. Compared with the test results, it shows that the developed model was capable of simulating the mechanical behaviour of LVL beam and glulam connection under load and/or fire with fairly good correlation.

With this model, it will allow structural designers to obtain the load-displacement curve of timber connections under force, fire or combination of the two. With this, key design parameters such as capacity, stiffness, displacement and ductility, which are required for seismic or fire design, can be obtained.

It is recommended that further verification and calibration of the model be conducted on various types of wood products, such as CLT, glulam, SCL and NLT, and fasteners, e.g. screw and rivet. Moreover, a database of the thermal and structural properties of the wood members and fasteners that are commonly used in timber constructions need to be developed to support and facilitate the application of the model.
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1 INTRODUCTION

FPInnovations has completed a survey with consultants and researchers on the use of analytical models and software packages related to the analysis and design of mass timber buildings. The responses identified that a lack of suitable models and related information for material properties of timber connections was creating an impediment to the design and construction of this type of buildings. Furthermore, there is currently a lack of computer models and expertise for carrying out performance-based design for wood buildings, in particular, seismic and/or fire performance design.

Wood is an anisotropic material. Its stiffness and strength properties vary as a function of grain orientation between the longitudinal, tangential, and radial directions. The failure modes and measured stress-strain relationships of wood depend on the direction of the load relative to the grain and the type of load (tension, compression or shear). The stress-strain relationships of wood in tension parallel or perpendicular to grain, and shear parallel or perpendicular to grain, are typically linear and the failure is brittle, while the stress-strain relationships of wood in compression parallel or perpendicular to grain are typically nonlinear and ductile. It is important to distinguish between the modes of failure because the effect of each mode on the ultimate strength of the timber connections may be quite different. So far the constitutive models in most finite element (FE) software packages can only deal with identical strength in compression and tension, and a single failure (yield/brittle) mode under all types of load. As a result, they are not suitable or optimal for wood products and timber connections.

With respect to the modelling of wood products or timber connections exposed to fire, thermal FE analysis has normally been used to calculate thermal field, taking into account the appropriate heat transfer mechanisms (conduction, radiation, etc.) as well as the thermo-physical properties of timber and steel at elevated temperatures (Maraveas et al. 2015). A mechanical analysis also can be performed on wood products or timber connections following the thermal analysis (Racher et al. 2010; Audebert et al. 2011, 2013 and 2014; Khelifa et al. 2014; Ohene 2014). However, without appropriate constitutive models for wood, it cannot accurately predict the failure modes of wood products or timber connections.

In this study, a sophisticated constitutive model for wood-based composite material under stress and temperature was developed. This constitutive model was programmed into a user-subroutine which can be added to most general-purpose finite element software. The developed model was validated with test results of a laminated veneer lumber (LVL) beam and a glulam bolted connection under force and/or fire.

2 OBJECTIVE

The objective of this project is to develop a sophisticated constitutive model for wood-based composite material which allows the prediction of load-displacement relationship of wood products and timber connections under force and/or fire loads. The developed model can be implemented in general FE software, therefore allowing structural designers to confidently
analyze and design structural systems with appropriate properties of wood products and timber connection.

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4 OVERVIEW OF COUPLED FIRE-STRUCTURAL MODELLING

Fire-structural modelling involves three fundamental components. They include (a) a fire model, (b) a heat transfer model, and (c) a structural model, as illustrated in Figure 1. The fire model generates a specific fire exposure (temperature boundary conditions) for the thermal simulation using the heat transfer model which in turn develops thermal gradients in the structural components / connections. The structural model will calculate the response of the components / connections under fire and loads (mechanical boundary conditions).

Figure 1. Flow chart for calculating structural fire-resistance (Buchanan 2002)
4.1 Fire Model

The compartment fire event generally has three distinct phases (Blagojevic and Pesic 2011): growth phase, steady-burning (or fully developed) phase, and decay phase, as illustrated in Figures 2 and 3.

![Figure 2. Idealized temperature-time phases of a well-ventilated compartment fire (Harmathy 1972)](image)

![Figure 3. Typical temperature history of gas contents of compartment in fire (Harmathy 1972)](image)

The fire model actually is a temperature-time curve depending on many parameters, e.g. fuel load density, compartment geometry, and ventilation factors. It is used to depict how the envelope temperature affecting the performance of the components / connections will change along with time during a fully developed compartment fire. The temperature-time curve can be a standard curve (ISO 834, ASTM E119 2016 or CAN/ULC S101), a measured curve from real
fire or a parametric fire curve (Dagenais 2016). The standard fire curve is traditionally used for evaluating materials and assemblies for code compliance.

**Figure 4.** Standard temperature-time curves

**Figure 5.** Parametric temperature-time curves
The standard CAN/ULC S101 temperature-time curve, as defined in Equation (1), is used in this study.

\[ T = 20 + 750 \left( 1 - e^{-0.49\sqrt{t}} \right) + 22\sqrt{t} \]  

where \( T \) [°C] is the temperature in the compartment at the time \( t \) [min].

4.2 Heat Transfer Model

Heat transfer is a thermal energy in transit due to a spatial temperature difference (Bergman et al. 2011). As such, whenever a temperature difference exists in a medium or between media, heat transfer must occur. It is classified into various mechanisms, such as thermal conduction, thermal convection and thermal radiation.

Conduction is the primary heat transfer mode occurring across a medium and satisfies Fourier’s law of heat conduction. Transient heat transfer within a solid material can be numerically expressed by the 3D partial differential equation shown in Equation (2).

\[
\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) + \dot{Q} = \rho c_s \left( k_x \frac{\partial T}{\partial x} \right)
\]

where \( T \) is the temperature [K]; \( k_i \) is thermal conductivities in \( i \) (x, y or z) directions [W/(m·K)]; \( \dot{Q} \) is the internally generated heat by the rate of heat consumption per unit volume due to chemical reaction (pyrolysis of wood) and the heat to evaporate water per unit volume [W/m³]; \( \rho \) is the density [kg/m³]; \( c_s \) is the specific heat [J/(kg·K)]; and \( t \) is the time [sec]. König & Walleij (1999) reported that the heat of reaction during pyrolysis can be neglected in heat transfer calculation. In an attempt to lower computational time, the internally generated heat from Equation (2) has therefore been neglected in the FE analysis developed herein. Steel is a non-combustible material and thus cannot generate heat at elevated temperatures; hence steel contribution to the internally generated heat has thereby also been ignored.

In the case of surfaces exposed to fire, heat is transferred from the emitting source through convection and thermal radiation. Convection typically refers to heat transfer occurring between a surface and a moving fluid when they are at different temperatures. Thermal radiation occurs when surfaces of the finite temperatures emit energy in the form of electromagnetic waves. The net emitted heat flux impinging on a surface exposed to fire is the sum of the emitted heat fluxes by convection and thermal radiation, as per Equation (3).

\[ q^* = q_{conv}^* + q_{rad}^* = \alpha_c(T_g - T_s) + \phi \varepsilon_m \varepsilon_f \sigma_{sb} (T_g^4 + T_s^4) \]

where \( \alpha_c \) is the convection heat transfer coefficient [W/(m²·K)]; \( T_g \) is the heating gas temperature [K]; \( T_s \) is the receiving surface temperature [K]; \( \phi \) is the view factor; \( \varepsilon_m \) is the surface material emissivity; \( \varepsilon_f \) is the fire source emissivity; \( \sigma_{sb} \) is the Stefan-Boltzmann constant, \( 5.67 \times 10^{-8} \text{ W/(m}^2\text{·K}^4) \). The temperature gradient generated by the heat transfer model depends on the convective and radiative heat transfer coefficients at the exposed surface as well as the heat conduction within the element. Therefore, for a material that has temperature-dependent
thermal properties (See Appendix I), proper properties are essential in predicting the temperature profile within the element.

4.3 Structural Model

Structural analysis is the determination of the effects of loads on structures and their components. It employs the fields of applied mechanics, materials science and applied mathematics to compute a structure's deformations, internal forces, stresses, support reactions etc. To perform an accurate analysis a structural engineer must determine such information as structural loads, geometry, support conditions, and materials properties. The results of such an analysis typically include support reactions, stresses and displacements. This information is then compared to criteria that indicate the conditions of failure. Advanced structural analysis may examine dynamic response, stability and non-linear behaviour.

There are three approaches to the analysis: the mechanics of materials approach (also known as strength of materials), the elasticity theory approach (which is actually a special case of the more general field of continuum mechanics), and the finite element approach. The first two approaches are generally made use of analytical formulations to simple linear elastic models, which lead to closed-form solutions and can often be solved by hand calculations. The finite element approach is a numerical method for solving differential equations generated by theories of mechanics such as elasticity theory and strength of materials. Regardless of the approach, the formulation is based on the same fundamental relationships: equilibrium, constitutive and compatibility. Though any existing finite element software packages can solve the equilibrium and compatibility equations, they have limited constitutive models, which are not suitable for anisotropic wood-based material.

Constitutive model, also called stress-strain relation, describes how the material behaves under various loads. An in-depth stress-strain relation includes elastic behaviour (e.g. Hooke's law), strength criterion (e.g. Hill theory), inelastic behaviour (yielding, plastic flow and hardening), and post-peak softening. It plays a critical role in the simulation of timber connections under forces and fire. A comprehensive constitutive model for wood-based material is developed. More information on the constitutive model is provided in Section 5.

4.4 Coupled Fire-Structural Analysis

Fire-structural analysis, also called thermal-stress/mechanical analysis, is to investigate the mechanical response of material or connections exposed to fire. Generally speaking, it includes sequentially coupled thermal-stress analysis and fully coupled thermal-stress analysis. If the stress/displacement solution is dependent on a temperature field and the temperature is not influenced by stress/displacement solution, a sequentially coupled thermal-stress analysis can be conducted by first performing the heat transfer analysis, then carrying the stress/displacement analysis with predefined temperature. A fully coupled temperature-displacement procedure is used to solve simultaneously the stress/displacement and the temperature fields. A fully coupled analysis is used when the thermal and mechanical solutions affect each other. The developed constitutive model in this study can be used in both coupled thermal-stress/mechanical analyses.
5 MATERIAL MODEL FOR WOOD-BASED PRODUCTS

Wood is a strong and versatile natural material. The orientation of the grains and the addition of growth rings give rise to three main mutually perpendicular axes namely Longitudinal (L), Radial (R) and Tangential (T) in any sawn timber, see Figure 6. The longitudinal axis lies parallel to the fibre or grain while both the radial and tangential axes are perpendicular to the direction of the grain. The main difference in the radial and tangential directions is their relative orientation to the growth rings; the radial axis is normal to the growth rings while the tangential axis is a tangent to the growth rings (Winandy, 1994).

![Diagram of wood axes](image)

Figure 6. Three principal axes of wood (Winandy, 1994)

The stiffness and strength properties of wood vary as a function of orientation between the longitudinal, tangential, and radial directions. Stiffness and strength are greatest in the longitudinal direction. The failure modes and measured stress-strain relationships of wood depend on the direction of the load relative to the grain and the type of load (tension, compression, or shear). The stress-strain relationships of wood in tension parallel or perpendicular to grain, and shear parallel or perpendicular to grain are typically linear and the failure is brittle, while the stress-strain relationships of wood in compression parallel or perpendicular to grain are typically nonlinear and ductile, as illustrated in Figure 7. It is important to distinguish between the modes of failure because the effect of each mode on the ultimate strength of the wood connections may be quite different. Temperature also affects the behaviour of wood.
Figure 7. Typical stress-strain curve of wood

Note: $\sigma_{io}$ is the axial strength in the $i$ direction [MPa]; $\sigma_{i0,T}$ and $\sigma_{i0,C}$ are the tensile and compressive strength in the $i$ direction [MPa]; $\sigma_{ij}$ is the shear strength in the $i-j$ plane [MPa]; $N_i$ and $n_i$ are parameters to determine initial and final ultimate yield surface, respectively; $\varepsilon_{Lo}$ is the initial damage strain for compression parallel to grain; $\varepsilon_{Ro}$ and $\varepsilon_{To}$ are the initial second-hardening strain for compression perpendicular to grain.

Existing material library in FE software packages typically does not have a constitutive model suitable for wood; thus a user subroutine needs to be developed. Influence of temperature on modulus of elasticity, shear modulus, strengths and the failure (brittle or ductile) mechanism will be considered.

5.1 Overview of Formulation

The wood constitutive model consists of a number of formulations that are merged together to form a comprehensive model:

- Elastic constitutive equation
- Strength criteria
- Post-peak softening
- Plastic flow and hardening
- Failure mode interaction
- Temperature influence

Each of these formulations is discussed separately in this section. The flowchart in Figure 8 shows how each formulation interacts with the others. This constitutive model was programmed into a user-subroutine, VUMAT, and added to general-purpose finite element software,
ABAQUS. A list of required parameters is given at the end of this section. Specific methods to derive mechanical properties for modelling input are thoroughly discussed in Appendix II.

5.2 Elastic Constitutive Equation

The elasticity of a material defines its deformation or strain response to applied stresses. Wood is a material with different elastic properties in longitudinal, radial and tangential directions. This makes wood a highly anisotropic material. Its elastic properties for design, analytical and
modelling purposes are often simplified in the form of orthotropic elasticity. If wood is considered as an orthotropic material, nine independent material parameters are needed to define its orthotropy, which include three modulus of elasticity ($E_L$, $E_R$ and $E_T$), three shear modulus ($G_{LR}$, $G_{LT}$ and $G_{RT}$) and three Poisson’s ratios ($\nu_{LR}$, $\nu_{LT}$ and $\nu_{RT}$). These parameters together define a constitutive relation for wood in the form of the three-dimensional generalized Hooke’s law.

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix} =
\begin{bmatrix}
D_{1111} & D_{1122} & D_{1133} & 0 & 0 & 0 \\
D_{1212} & D_{1222} & D_{1233} & 0 & 0 & 0 \\
D_{1313} & D_{2323} & D_{2333} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{1212} & 0 & 0 \\
0 & 0 & 0 & 0 & D_{2323} & 0 \\
0 & 0 & 0 & 0 & 0 & D_{3131}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{23} \\
\varepsilon_{31}
\end{bmatrix}
\]

(10)

Note: $\sigma_{ij}$ and $\varepsilon_{ij}$ are the stress and strain, respectively.

\[
D_{1111} = E_L(1 - \nu_{RT}\nu_{TR}) \\
D_{2222} = E_R(1 - \nu_{LT}\nu_{TL}) \\
D_{3333} = E_T(1 - \nu_{LR}\nu_{RL}) \\
D_{1122} = E_L(\nu_{RL} + \nu_{TL}\nu_{TR}) \\
D_{1133} = E_L(\nu_{TL} + \nu_{RL}\nu_{TR}) \\
D_{2233} = E_R(\nu_{TR} + \nu_{LR}\nu_{TL}) \\
D_{1212} = 2G_{LR} \\
D_{2323} = 2G_{RT} \\
D_{3131} = 2G_{LT} \\
\varepsilon_{12} = \gamma_{LR}/2 \\
\varepsilon_{23} = \gamma_{RT}/2 \\
\varepsilon_{31} = \gamma_{TL}/2
\]

\[
\tau = \frac{1}{1 - \nu_{LR}\nu_{RL} - \nu_{RT}\nu_{TR} - \nu_{TL}\nu_{LT} - 2\nu_{RL}\nu_{TR}\nu_{L}}
\]

The Poisson’s ratios are related as follows

\[
\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}
\]

(10)

The two subscripts, $i$ and $j$ stand for the transverse strain in the $j$-direction, when the material is stressed in the $i$-direction.
Because the wood model is orthotropic, the orientation of the wood specimen must be set relative to the global coordinate system of ABAQUS. The orthotropic constitutive relationships of the wood material are developed in the material coordinate system (i.e., the longitudinal and radial and tangential directions). The user must define the orientation of the material coordinate system with respect to the global coordinate system. Appropriate coordinate transformations are formulated in ABAQUS between the material and global coordinate systems. Such coordinate transformations are necessary because any differences between the grain axis and the structure axis can have a great impact on the structural response.

### 5.3 Strength Criterion

For wood, the strength is significantly different in the longitudinal, radial and tangential directions. The strength in the longitudinal direction is greater than that in the radial and tangential directions. Radial and tangential strengths are generally similar; hence wood is normally referred to as transversely isotropic. Longitudinal strength is popularly known as the parallel-to-grain strength whereas radial and tangential strengths are generally categorised as the perpendicular-to-grain strength (Winandy 1994).

Various strength criteria (Nahas 1987) have been developed for predicting localized material failure due to stress caused by static load. Criteria are usually applied in terms of a stress space or a strain space. Strain-based criteria were not evaluated because failure strains are not reported in the literature for wood. One cannot derive failure strains from stresses if the stress-strain behaviour is nonlinear, as it is for wood in compression. For stress-based criteria, the stresses are transformed to the principal material axes (L-R-T axes) before application of the strength criteria. A brief summary of some typical strength criteria is given here and a comparison of the criteria is given in Table 1:

- **Maximum Stress (commonly applied limit theory):** This is one of the most common limit theories. Failure occurs when any component of stress exceeds its corresponding strength.

- **Coulomb-Mohr (Coulomb 1773; Mohr 1900):** This is the most common strength criterion encountered in geotechnical engineering. It is used to determine the failure load as well as the angle of fracture in concrete and similar materials. Coulomb’s friction hypothesis is used to determine the combination of shear and normal stress that will cause a fracture of the material. Mohr’s circle is used to determine which principal stresses that will produce this combination of shear and normal stress, and the angle of the plane in which this will occur. According to the principle of normality the stress introduced at failure will be perpendicular to the line describing the fracture condition. Generally the theory applies to materials for which the compressive strength far exceeds the tensile strength.

- **Von Mises (maximum distorted energy, 1913):** In this theory, a ductile material under a general stress state yields when its shear distortional energy reaches the criterial shear distortional energy under simple tension.
• **Tresca** (maximum shear stress, 1864): The material remains elastic when all three principal stresses are roughly equivalent (a hydrostatic pressure), no matter how much it is compressed or stretched. However, when one of the principal stresses becomes smaller (or larger) than the others the material is subject to shearing. In such situations, if the shear stress reaches the yield limit then the material enters the plastic domain. This is a special case for Coulomb-Mohr criterion with the coefficient of internal friction is equal to zero (Christensen 2016).

• **Hill** (1950): This theory is a generalization of von Mises maximum distortional energy theory to include anisotropic materials. It considers “interaction” between the failure strengths, thus a smooth failure envelope rather than the intersecting straight lines.

• **Tsai-Wu** (tensor polynomial theory that was originally developed for anisotropic material, 1971): it contains linear and quadratic stress terms. Seven coefficients must be defined for transversely isotropic applications. The non-interaction coefficients that contain one component of stress are determined from measured uniaxial and pure shear strengths. The interaction terms that have two or more components of stress multiplied together are determined from measured biaxial strengths.

• **Hoffman** (1967): Hoffman extended Hill’s distortional energy criterion for orthotropic materials to account for different strengths in tension and compression. The criterion contains linear and quadratic stress terms. Six coefficients are determined from uniaxial stress and pure shear tests. Biaxial strengths are not needed.

• **Norris** (1962): Norris developed three yield criteria for mutually orthogonal planes. Each criterion contains quadratic stress terms (no linear terms). Nine coefficients are determined from uniaxial and pure shear tests. Tensile strengths are used when the corresponding stresses are tensile. Compressive strengths are used when the corresponding stresses are compressive.

• **Hashin** (1980): Hashin formulated a quadratic stress polynomial in terms of the invariants of a transversely isotropic material. Separate formulations are identified for longitudinal, radial and tangential modes by assuming that failure is produced by the normal and shear stresses acting on the failure plane. In addition, the longitudinal, radial and tangential modes are subdivided into tensile and compressive modes. Assumptions include: (1) biaxial compressive strength perpendicular to the grain is much greater than the uniaxial compressive strength and (2) shear stress does not contribute to compressive failure parallel to the grain. All coefficients are determined from six uniaxial and shear strengths.

• **Extended Yamada-Sun** (1978): Three yield criteria are reported for mutually orthogonal planes each criterion predicts that the normal and shear stresses are
mutually weakening. Nine coefficients are determined from uniaxial and pure shear tests.

### Table 1. Comparison of various strength criteria

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Ductile or brittle material</th>
<th>Interaction between the strengths or not?</th>
<th>Can failure mode be judged?</th>
<th>Account for different strengths in tension and compression?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Stress</td>
<td>D &amp; B</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Coulomb-Mohr</td>
<td>B</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Von Mises</td>
<td>D</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Tresca</td>
<td>D</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Hill</td>
<td>D</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Tsai-Wu</td>
<td>D &amp; B</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Hoffman</td>
<td>D &amp; B</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Norris</td>
<td>D &amp; B</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Hashin</td>
<td>D &amp; B</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Extended Yamada-Sun</td>
<td>D &amp; B</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The von Mises and Tresca strength (yield) criteria apply to very ductile isotropic metals, but function poorly for all other materials types. The Coulomb-Mohr criterion is suitable for brittle materials. The Hill strength (yield) criteria can be used for orthotropic materials, however, it is not directly applicable to wood materials because it does not model different strengths in tension and compression. The maximum stress does not include the interaction between the strengths. The Tsai-Wu and Hoffman interactive strength criteria predict when a given set of stresses will produce failure, but they do not predict the mode of failure. The Norris approach is able to predict the failure in orthogonal planes rather than orthogonal axes. Both Hashin and extended Yamada-Sun approaches can predict four independent models of failure: tensile and compressive failure parallel to the grain and tensile and compressive failure perpendicular to the grain.

Since the Extended Yamada-Sun is simple and effective approach in predicting failure in both the deterministic and probabilistic sense (Nahas 1987), it was adopted in this study. According to this approach, the strength criterion used in this study is shown below.
a) Longitudinal:

\[
\frac{\sigma_{11}^2}{\sigma_{Lo}^2} + \frac{\sigma_{12}^2}{\sigma_{Kro}^2} + \frac{\sigma_{13}^2}{\sigma_{Kro}^2} \leq 1 \quad \sigma_{Lo} = \begin{cases} 
\sigma_{Lo,T} & \text{if } \sigma_{11} \geq 0, \text{ Brittle} \\
\sigma_{Lo,C} & \text{if } \sigma_{11} < 0, \text{ Yield}
\end{cases}
\]  

(11a)

b) Radial:

\[
\frac{\sigma_{22}^2}{\sigma_{Ro}^2} + \frac{\sigma_{23}^2}{\sigma_{Kro}^2} + \frac{\sigma_{23}^2}{\sigma_{Kro}^2} \leq 1 \quad \sigma_{Ro} = \begin{cases} 
\sigma_{Ro,T} & \text{if } \sigma_{22} \geq 0, \text{ Brittle} \\
\sigma_{Ro,C} & \text{if } \sigma_{22} < 0, \text{ Yield}
\end{cases}
\]  

(11b)

c) Tangential:

\[
\frac{\sigma_{33}^2}{\sigma_{To}^2} + \frac{\sigma_{31}^2}{\sigma_{Kro}^2} + \frac{\sigma_{32}^2}{\sigma_{Kro}^2} \leq 1 \quad \sigma_{To} = \begin{cases} 
\sigma_{To,T} & \text{if } \sigma_{33} \geq 0, \text{ Brittle} \\
\sigma_{To,C} & \text{if } \sigma_{33} < 0, \text{ Yield}
\end{cases}
\]  

(11c)

where \( \sigma_{io} \) is the axial strength in the \( i \) direction [MPa]; \( \sigma_{io,T} \) and \( \sigma_{io,C} \) are the tensile and compressive strength in the \( i \) direction [MPa]; \( \sigma_{ij} \) is the shear strength in the \( i-j \) plane [MPa].

Once the stresses meet the strength criterion in any direction, brittle failure or ductile yield will occur depending on the stress components. If the wood is subjected to tensile stress and/or shear stress, it will fail in a brittle way which will be simulated using post-peak softening; otherwise, the wood will yield with plastic flow and hardening. With respect to wood under compression perpendicular to grain, second hardening will happen when the strain reaches a specific criterion.

5.4 Post-Peak Softening

Post-peak softening occurs in the tensile and shear modes of wood. Continuum Damage Mechanics (CDM) (Matzenmiller, Lubliner and Taylor 1995) was adapted to model degradation with a damage formulation in this study. A scalar damage parameter, \( d_i \), \( 0 \leq d_i \leq 1 \) transforms the stress tensor associated with the undamaged state, \( \bar{\sigma}_{ij} \), into the stress tensor associated with the damaged state, \( \sigma_{ij} \):

\[
\sigma_{ij} = (1 - d_i)\bar{\sigma}_{ij}
\]  

(12)

The damage parameter ranges from zero for no damage to approaching unity for maximum damage. Thus, \( 1 - d_i \) is a reduction factor associated with the amount of damage. Two advantages can be obtained by using this formulation: (1) stiffness is degraded in conjunction with strength; and (b) progressive softening is dependent of subsequent loading. Figure 9 shows how the damage parameter affects the stress-strain curve.
Damage formulations are typically based on strain, stress, or energy. An exponential damage evolution was proposed by Chen (2011) based on damage accumulation on the history of strains:

\[ d_i = 1 - \exp \left( \frac{-\left( \tau_{i,0} - \tau_i \right)}{\tau_{i,0}} \right) \quad (12) \]

Where \( \tau_{i,0} \) and \( \tau_i \) are the undamaged elastic strain energy norms at the time when stresses meet the strength criterion in the \( i \) direction and beyond, respectively. A famous strain-based theory by Simo and Ju (1987) was adopted to calculate the damage based on the total strains and the undamaged modulus of elasticity. Assuming that the damage parameters in the three major directions are independent of each other, the undamaged elastic strain energy norms, \( \tau_i \), can be calculated by:

a) **Longitudinal**

\[ \tau_L = \sqrt{\sigma_{11}^* \varepsilon_{11}^* + 2(\sigma_{12}^* \varepsilon_{12}^* + \sigma_{31}^* \varepsilon_{31}^*)} \quad (13a) \]

b) **Radial**

\[ \tau_R = \sqrt{\sigma_{22}^* \varepsilon_{22}^* + 2(\sigma_{12}^* \varepsilon_{12}^* + \sigma_{23}^* \varepsilon_{23}^*)} \quad (13b) \]

c) **Tangential**

\[ \tau_T = \sqrt{\sigma_{33}^* \varepsilon_{33}^* + 2(\sigma_{31}^* \varepsilon_{31}^* + \sigma_{23}^* \varepsilon_{23}^*)} \quad (13c) \]

where \( \sigma_{ij}^* \) is undamaged stress [MPa].

Another issue is strength coupling, in which degradation in one direction affects degradation in another direction. If failure occurs in the longitudinal modes, then all six stress components are degraded uniformly. This is because longitudinal failure is catastrophic and will render the wood useless. The wood is not expected to carry load in either the longitudinal or radial and tangential directions once the wood fibres are broken. If failure occurs in the radial and tangential modes,
then only the radial and tangential stress components are degraded. This is because radial and tangential failure is not catastrophic (the wood is expected to continue to carry the load in the longitudinal direction). Based on these assumptions, the following degradation model is implemented:

\[
\begin{align*}
    d_L &= d(\tau_L) \quad (14a) \\
    d_R &= \max[d(\tau_L), d(\tau_R)] \quad (14b) \\
    d_T &= \max[d(\tau_L), d(\tau_T)] \quad (14c) \\
    \sigma_{11} &= (1 - d_L)\bar{\sigma}_{11} \quad (15a) \\
    \sigma_{22} &= (1 - d_R)\bar{\sigma}_{22} \quad (15b) \\
    \sigma_{33} &= (1 - d_T)\bar{\sigma}_{33} \quad (15c) \\
    \sigma_{12} &= (1 - d_L)\bar{\sigma}_{12} \quad (15d) \\
    \sigma_{23} &= [1 - \max(d_R, d_T)]\bar{\sigma}_{23} \quad (15e) \\
    \sigma_{31} &= (1 - d_L)\bar{\sigma}_{31} \quad (15f)
\end{align*}
\]

### 5.5 Plastic Flow and Hardening

In compression parallel to grain, wood cells deform along the longitudinal direction and are shortened by compressive stresses (Winandy 1994). A typical stress-strain relationship (Figure 7) shows a ductile behaviour consisting of an initial linear elastic relationship, followed by a distinct plastic behaviour. Stresses in compression perpendicular-to-grain increase until fibres fold together with little or no cavity in-between them (Winandy 1994). The peak compressive capacity of wood perpendicular to grain is normally significantly less in this case compared to compression parallel to grain. A more stable post-peak compressive stress is obtained as compared to the compression parallel-to-grain. Notwithstanding, the overall ductility of wood is much higher in the longitudinal direction as opposed to the radial and tangential direction.

The yield of wood under compression either parallel or perpendicular to grain will occur once the strength criterion is satisfied, following the plastic flow and hardening rule. Second hardening will happen once the compression strain perpendicular to grain reaches a specific strain criterion.

#### 5.5.1 Plastic Flow

The plasticity algorithms limit the stress components once the strength criterion is satisfied. This is done by returning the stress state back to the yield surface. Traditional approach for modelling plasticity is to partition the stress and strain tensors into elastic and plastic parts.

\[
\Delta \varepsilon_{ij} = \Delta \varepsilon_{ij}^e + \Delta \varepsilon_{ij}^p = \Delta \varepsilon_{ij}^e + \Delta \lambda \frac{\partial g}{\partial \sigma_{ij}}
\]  

\[
(16)
\]
where $\Delta \varepsilon_{ij}$, $\Delta \varepsilon_{ij}^e$ and $\Delta \varepsilon_{ij}^p$ are total, elastic and plastic strain increment; $\Delta \lambda$ is a positive scalar of proportionality; $g$ is a plastic potential function.

Partitioning is done with return mapping algorithms that enforce the plastic consistency conditions. Such algorithms allow one to control plastic strain generation. In addition, return mapping algorithms with associated flow satisfy the second law of hemodynamics. In this study, radial-return algorithm (Lubliner 2008) was adopted. Therefore, the plastic potential function, $g$, can be simplified to yield function:

a) Longitudinal:

$$f_L(\sigma_{11}, \sigma_{12}, \sigma_{31}) = \frac{\sigma_{11}^2}{\sigma_{11}^{\text{YLC}}} + \frac{\sigma_{12}^2}{\sigma_{12}^{\text{YLC}}} + \frac{\sigma_{31}^2}{\sigma_{31}^{\text{YLC}}} - 1 \quad (17a)$$

b) Radial:

$$f_R(\sigma_{22}, \sigma_{12}, \sigma_{23}) = \frac{\sigma_{22}^2}{\sigma_{22}^{\text{YLC}}} + \frac{\sigma_{12}^2}{\sigma_{12}^{\text{YLC}}} + \frac{\sigma_{23}^2}{\sigma_{23}^{\text{YLC}}} - 1 \quad (17b)$$

c) Tangential:

$$f_T(\sigma_{33}, \sigma_{31}, \sigma_{23}) = \frac{\sigma_{33}^2}{\sigma_{33}^{\text{YLC}}} + \frac{\sigma_{31}^2}{\sigma_{31}^{\text{YLC}}} + \frac{\sigma_{23}^2}{\sigma_{23}^{\text{YLC}}} - 1 \quad (17c)$$

The positive scalar of proportionality $\Delta \lambda$ by enforcing the plastic consistency condition. The condition is expressed as:

$$\Delta f = f^{n+1} - f^n = 0 \quad (18)$$

$f^n$ and $f^{n+1}$ are the yield surface function at time increments $n$ to $n+1$. The stress state at the beginning of the time step lies on the yield surface, thus, $f^n = 0$. The stress state at the end of the time step is returned to the yield surface by the plasticity algorithm, thus, $f^{n+1} = 0$. Therefore, $\Delta f = 0$. The solution of the consistency condition in the above equation determines $\Delta \lambda$, which, in turn, determines the partitioning of the total strain rate into elastic and plastic components. The stresses are updated from the elastic strain components. Separate plasticity algorithms are proposed for the modes in the three major directions by enforcing separate consistency conditions. Each consistency condition is derived below.

For longitudinal direction, a first-order Taylor series expansion of the consistency condition $\Delta f_L = f_L^{n+1} - f_L^n = 0$ from time increment $n$ to $n+1$ gives:

$$\frac{\partial f_L}{\partial \sigma_{11}} \Delta \sigma_{11} + \frac{\partial f_L}{\partial \sigma_{12}} \Delta \sigma_{12} + \frac{\partial f_L}{\partial \sigma_{31}} \Delta \sigma_{31} = 0 \quad (19)$$

Expansion of the stress increments gives:

$$\Delta \sigma_{11} = \Delta \sigma_{11}^e - D_{1111} \frac{\partial f_L}{\partial \sigma_{11}} |_{n} \Delta \lambda_L \quad (20a)$$
\[ \Delta \sigma_{12} = \Delta \sigma_{12}^* - D_{1212} \frac{\partial f_L}{\partial \sigma_{12}} \Delta \lambda_L \]  
\[ \Delta \sigma_{31} = \Delta \sigma_{31}^* - D_{3131} \frac{\partial f_L}{\partial \sigma_{31}} \Delta \lambda_L \]  
(20b)  
(20c)

Substitution of the updates from Eq.(20) into the consistency condition in Eq.(19) results in the following expression for \( \Delta \lambda_L \):

\[ \Delta \lambda_L = \left( \frac{\frac{\partial f_L}{\partial \sigma_{11}} \Delta \sigma_{11}^* + \frac{\partial f_L}{\partial \sigma_{12}} \Delta \sigma_{12}^* + \frac{\partial f_L}{\partial \sigma_{31}} \Delta \sigma_{31}^*}{D_{1111} \frac{\sigma_{11}}{\sigma_{11}} + 2D_{1212} \frac{\sigma_{12}}{\sigma_{12}} + 2D_{3131} \frac{\sigma_{31}}{\sigma_{31}}} \right) \]  
(21)

The expression in the numerator is recognized as the first-order Taylor series expansion of \( f_L^* \), where \( f_L^* = f_L(\sigma_{11}^*, \sigma_{12}^*, \sigma_{31}^*) \) is the value of the failure criterion calculated from the trail elastic stresses. Therefore, the expression for \( \Delta \lambda_L \) reduces to:

\[ \Delta \lambda_L = \frac{f_L(\sigma_{11}^*, \sigma_{12}^*, \sigma_{31}^*)/4}{D_{1111} \frac{\sigma_{11}}{\sigma_{11}} + 2D_{1212} \frac{\sigma_{12}}{\sigma_{12}} + 2D_{3131} \frac{\sigma_{31}}{\sigma_{31}}} \]  
(21)

Similarly, the parameters, \( \Delta \lambda_R \) and \( \Delta \lambda_T \), in the radial and tangential directions can be derived as follows:

\[ \Delta \lambda_R = \frac{f_R(\sigma_{22}^*, \sigma_{12}^*, \sigma_{23}^*)/4}{D_{2222} \frac{\sigma_{22}}{\sigma_{22}} + 2D_{1212} \frac{\sigma_{12}}{\sigma_{12}} + 2D_{2323} \frac{\sigma_{23}}{\sigma_{23}}} \]  
(22)

\[ \Delta \lambda_T = \frac{f_T(\sigma_{33}^*, \sigma_{11}^*, \sigma_{23}^*)/4}{D_{3333} \frac{\sigma_{33}}{\sigma_{33}} + 2D_{3131} \frac{\sigma_{31}}{\sigma_{31}} + 2D_{2323} \frac{\sigma_{23}}{\sigma_{23}}} \]  
(23)

### 5.5.2 Hardening

Wood exhibits pre-peak nonlinearity in compression parallel and perpendicular to grain. Radial and tangential hardening was previously demonstrated in Figure 7. A translating yield-surface approach that simulates a gradual change in modulus was adopted in this study. The approach is to define initial yield surfaces that harden (translate) until they coincide with the ultimate yield surfaces, as demonstrated in Figure 10 for the longitudinal model. The initial location of the yield surface determines the onset of plasticity. The rate of translation determines the extent of nonlinearity.
The ultimate and initial yield surfaces are described in Eqs. (17) and (24), respectively.

a) Longitudinal:

\[
f_{L,i}(\sigma_{11}, \sigma_{12}, \sigma_{31}) = \frac{\sigma_{11}^2}{\sigma_{L_{o},c}^2(1-N_{L})^2} + \frac{\sigma_{12}^2}{\sigma_{L_{o},T}^2} + \frac{\sigma_{31}^2}{\sigma_{L_{o},c}^2} - 1
\]  

(24a)

b) Radial:

\[
f_{R,i}(\sigma_{22}, \sigma_{12}, \sigma_{23}) = \frac{\sigma_{22}^2}{\sigma_{R_{o},c}^2(1-N_{R})^2} + \frac{\sigma_{12}^2}{\sigma_{R_{o},T}^2} + \frac{\sigma_{23}^2}{\sigma_{R_{o},c}^2} - 1
\]  

(24b)

c) Tangential:

\[
f_{T,i}(\sigma_{33}, \sigma_{31}, \sigma_{23}) = \frac{\sigma_{33}^2}{\sigma_{T_{o},c}^2(1-N_{T})^2} + \frac{\sigma_{31}^2}{\sigma_{T_{o},T}^2} + \frac{\sigma_{23}^2}{\sigma_{T_{o},c}^2} - 1
\]  

(24c)

where \( N_i \) is a parameter to determine the initial yield surface.

The state variable that defines the translation of the yield surface is known as the backstress and is denoted by \( \alpha_{ij} \). Prepeak nonlinearity is modeled in compression only, therefore only backstress in the longitudinal direction, \( \alpha_{11} \), is required. The value of the backstress is “0” upon initial yield and \( “N_i X_c” \) at ultimate yield (in uniaxial compression). The maximum backstress occurs at ultimate yield and is equal to the total translation of the yield surface in stress space. Kinematic hardening rule (Lubliner 2008) was adopted to define the growth of the backstress based on the stress and plastic strain in this study. This is accomplished by defining the incremental backstress:

a) Longitudinal:

\[
\Delta \alpha_L = C_{a,L} G_{a,L}(\sigma_{11} - \alpha_L)\Delta \varepsilon_L
\]  

(25a)
b) Radial:

\[ \Delta \sigma_R = C_{a,R} G_{a,R} (\sigma_{22} - \alpha_R) \Delta \varepsilon_R \]  

(25b)

c) Tangential:

\[ \Delta \sigma_T = C_{a,T} G_{a,T} (\sigma_{33} - \alpha_T) \Delta \varepsilon_T \]  

(25c)

where \( \Delta \sigma_i \) and \( \Delta \varepsilon_i \) are the increment of backstress and strain, respectively, in the \( i \) direction; \( C_{a,i} \) and \( G_{a,i} \) are hardening parameters in the \( i \) direction. The parameter \( C_{a,i} \) determines the rate of hardening (i.e., how rapidly or gradually the stress-strain curve harden). The value of \( C_{a,i} \) needed for any particular fit depends on the properties of the material being modeled. It is selected by running simulations with various values of \( C_{a,i} \) and comparing those simulations with data. The function \( G_{a,i} \) restricts the motion of the yield surface so that it cannot translate outside the ultimate surface (Sandler, DiMaggio and Barron 1984). Each function is derived from the yield surface definition and hardening stress update.

For longitudinal direction, the desired attributes of the limiting function are \( G_{a,L} = 1 \) at initial yield and \( G_{a,L} = 0 \) at ultimate yield. Hardening is modeled in compression, but not shear or tension, so the only stress component with hardening is \( \sigma_{11} \). The initial yield strength in compression is defined as \( \tilde{\sigma}_L \), and the ultimate strength in compression is defined as \( \sigma_L^F \). The relationship between these strengths is:

\[ \tilde{\sigma}_L = \sigma_L^F (1 - N_L) \]  

(26)

For combined stress states, the ultimate yield strength from Eq.(17a) is:

\[ \sigma_L^F = \sigma_{Lo,C} \sqrt{1 - \left( \frac{\sigma_{11}^2}{\sigma_{Lo,C}^2} + \frac{\sigma_{22}^2}{\sigma_{Lo,C}^2} \right)} \]  

(27)

For the case of uniaxial compressive stress, the ultimate yield strength is \( \sigma_L^F = \sigma_{Lo,C} \). The longitudinal stress update with hardening is:

\[ \sigma_{11} = \tilde{\sigma}_L + \alpha_L \]  

(28)

At ultimate yield, this relationship becomes:

\[ \sigma_L^F = \tilde{\sigma}_L + \alpha_L^{max} \]  

(29)

Substitution of Eq.(29) into Eq.(26) and rearranging gives:

\[ 1 - \frac{\alpha_L^{max}}{N_L \sigma_L^F} = 0 \]  

(30)

The above function has the desired attribute in that it equals zero when the stress state lines on the ultimate yield surface. Thus, one defines:

\[ G_{a,L} = 1 - \frac{\alpha_L}{N_L \sigma_L^F} \]  

(31)
The value of the limiting function is $G_{\alpha,L} = 1$ at initial yield because $\alpha_L = 1$ at initial yield. The value of the limiting function is $G_{\alpha,L} = 0$ at ultimate yield because $\alpha_L = \alpha_{L}^{max}$ from Eq.(30). Thus, $G_{\alpha,L}$ limits the growth of the backstress as the ultimate yield surface is approached. Similarly, the limiting functions $G_{\alpha,R}$ and $G_{\alpha,T}$ can be defined as:

$$G_{\alpha,R} = 1 - \frac{\alpha_R}{N_R\sigma_F^R}$$  \hspace{1cm} (32)
$$G_{\alpha,T} = 1 - \frac{\alpha_T}{N_T\sigma_F^T}$$  \hspace{1cm} (33)

The corresponding ultimate yield strengths from Eq.(17b, c) are:

$$\sigma_R^F = \sigma_{R0,C} \sqrt{1 - \left( \frac{\sigma_{12}^F}{\sigma_{LR_0}^F} + \frac{\sigma_{23}^F}{\sigma_{RT_0}^F} \right) \cdot \left( \frac{1}{\rho} \right)}$$  \hspace{1cm} (34)
$$\sigma_T^F = \sigma_{T0,C} \sqrt{1 - \left( \frac{\sigma_{21}^F}{\sigma_{LT_0}^F} + \frac{\sigma_{32}^F}{\sigma_{RT_0}^F} \right) \cdot \left( \frac{1}{\rho} \right)}$$  \hspace{1cm} (35)

The effective strain increment $\Delta \varepsilon_i$ is equal to

a) Longitudinal

$$\Delta \varepsilon_L = \sqrt{\Delta \varepsilon_{11}^2 + 2(\Delta \varepsilon_{12}^2 + \Delta \varepsilon_{21}^2)}$$  \hspace{1cm} (36a)

b) Radial

$$\Delta \varepsilon_R = \sqrt{\Delta \varepsilon_{23}^2 + 2(\Delta \varepsilon_{12}^2 + \Delta \varepsilon_{23}^2)}$$  \hspace{1cm} (36b)

c) Tangential

$$\Delta \varepsilon_T = \sqrt{\Delta \varepsilon_{33}^2 + 2(\Delta \varepsilon_{31}^2 + \Delta \varepsilon_{23}^2)}$$  \hspace{1cm} (36c)

### 5.5.3 Second Hardening

When the compressive strain perpendicular to grain reaches a criterion, second hardening will occur, Figure 11. Similar to the hardening, the translating yield-surface approach was also used for second hardening, in which the ultimate yield surface will harden (translate) to final ultimate yield surface (Eq.(37)).
where \( n_i \) is parameter to determine the final ultimate yield surface. Similarly, an incremental second backstress, \( \Delta \beta_i \) is defined:

a) Radial:

\[
\Delta \beta_R = C_{\beta,R} G_{\beta,R}(\sigma_{22} - \alpha_R - \beta_R)\Delta \epsilon_R \quad (381)
\]

b) Tangential:

\[
\Delta \beta_T = C_{\beta,T} G_{\beta,T}(\sigma_{33} - \alpha_T - \beta_T)\Delta \epsilon_T \quad (38b)
\]

where \( C_{\beta,i} \) and \( G_{\beta,i} \) are second hardening parameters in the \( i \) direction. The parameter \( C_{\beta,i} \) determines the rate of second hardening. The value of \( C_{\beta,i} \) needed for any particular fit depends on the properties of the material being modeled. It is selected by running simulations with various values of \( C_{\beta,i} \) and comparing those simulations with data. The function \( G_{\beta,i} \) restricts the motion of the yield surface so that it cannot translate outside the final ultimate surface and can be defined as:

\[
G_{\beta,R} = 1 - \frac{\beta_R}{(n_R - 1)\sigma_R} \quad (39)
\]
\[ G_{\beta,T} = 1 - \frac{\beta_T}{(n_T-1)\sigma_T} \]  \hfill (40)

### 5.6 Temperature Influence

In general, the mechanical properties of wood decrease when heated and increase when cooled (Forest Products Laboratory 2010). The change in properties that occurs when wood is quickly heated or cooled and then tested at that condition is termed an immediate effect. At temperatures below 100°C, the immediate effect is essentially reversible; that is the property will return to the value at the original temperature if the temperature change is rapid. In addition to the reversible effect of temperature on wood, there is an irreversible effect at elevated temperature. This permanent effect is one of degradation of wood substance, which results in loss of weight and strength. The loss depends on factors that include moisture content, heating medium, temperature, exposure period, and to some extent, species and size of piece involved. The temperature effect during most fire events follows in this irreversible effect category. Thus, the irreversible effects of temperature will be studied in this study.

Many different proposals can be found in literature for the temperature dependent thermal properties of timber (Buchanan 2001; Cachim and Franssen 2009; Frangi 2001; Hopkin et al. 2011; König and Walleij 1999; König 2006). However, to be in line with current good practice EN1995-1-2 relationships are adopted for thermo-physical (conductivity, specific heat and density, see Appendix I) and mechanical (modulus of elasticity, shear modulus and strength, discussed below) properties. Such relationship account implicitly for the complex physical and chemical phenomena, so that a simple conductive heat transfer analysis and thermal-mechanical analysis can be carried out without requiring many of the physical complexities of timber combustion and charring to be specifically modelled. Thus effects like moisture migration, transient thermal creep, formation of char, shrinking and cracking of charcoal are represented by adjusted “effective values” rather than using real measured material properties. To use more realistic values requires the consideration of more complicated algorithms within the simulation, such as thermal transport by mass flow (moisture movement), the constantly changing geometry, and the formation of cracks in the charcoal introduced by thermal stresses. The complexity of these problems leads to a huge input effort, coupled simulations and long calculation time which the user would normally want to avoid.

EN1995-1-2 specifies that the local values of strength and modulus of elasticity parallel to grain for softwood should be multiplied by a temperature dependent reduction factor according to Eqs.(41) to (45) and Figure 12.

\[ k_{E,t} = \begin{cases} 1 - \frac{1}{160}(T - 20) & 20 ^\circ C \leq T < 100 ^\circ C \\ 0.5 - \frac{1}{400}(T - 100) & 100 ^\circ C \leq T \leq 300 ^\circ C \end{cases} \]  \hfill (41)

\[ k_{E,c} = \begin{cases} 1 - \frac{13}{1600}(T - 20) & 20 ^\circ C \leq T < 100 ^\circ C \\ 0.35 - \frac{7}{4000}(T - 100) & 100 ^\circ C \leq T \leq 300 ^\circ C \end{cases} \]  \hfill (42)
where $k_{E,t}$, $k_{E,c}$, $k_{t,para}$, $k_{c,para}$, and $k_{s,para}$ are the temperature reduction factor for tensile and compressive modulus of elastic, tensile, compressive and shear strength parallel to grain.

(a)
Figure 12. Effect of temperature on modulus of elasticity (a) and strength (b) of softwood parallel to grain according to Eqs. (41) to (45)

According to EN1995-1-2, the same reduction of strength as for compression parallel to grain may be applied to compression perpendicular to grain; for shear with both stress components perpendicular to grain (rolling shear), the same reduction of strength may be applied as for compression parallel to grain. Based on the test results by Gerhards (1982), it is reasonable to apply the same reduction of modulus of elasticity parallel to grain to modulus of elasticity perpendicular to grain, shear modulus parallel and perpendicular to grain, apply the same reduction of compressive strength parallel to grain to tensile and shear strength perpendicular to grain.

A four-segment curve model, Eqs. (46) and (47), is used to describe the influence of temperature on the modulus of elasticity and strength of wood. It assumes that (a) the mechanical properties of wood are kept constant when the temperature is equal to or less than a standard temperature, \( T_1 \); (b) the mechanical properties of wood will decrease to zero once the temperature reaches, \( T_1 \), however, a very small value will be used instead of zero due to convergent problems; (c) the mechanical properties of wood will change along the two lines among \( T_1, T_2 \) and \( T_3 \). The parameters of temperature and reduction parameters are given in Table 2.

\[
k_E = \begin{cases} 
  k_{E1} & T \leq T_1 \\
  k_{E1} + \frac{T-T_1}{T_2-T_1}(k_{E2} - k_{E1}) & T_1 < T \leq T_2 \\
  k_{E2} + \frac{T-T_2}{T_3-T_2}(k_{E3} - k_{E2}) & T_2 < T \leq T_3 \\
  k_{E3} & T_3 < T
\end{cases}
\]  

\[(46)\]
\[
T_e = \begin{cases} 
k_{S_1} & T \leq T_1 \\
k_{S_1} + \frac{T-T_1}{T_2-T_1}(k_{S_2} - k_{S_1}) & T_1 < T \leq T_2 \\
k_{S_2} + \frac{T-T_2}{T_3-T_2}(k_{E_3} - k_{S_2}) & T_2 < T \leq T_3 \\
k_{S_3} & T_3 < T 
\end{cases}
\] (47)

where \( k_E \) and \( k_S \) are the temperature reduction factor for modulus of elastic and strength; \( k_{E_1} \) and \( k_{S_1} \) are the temperature reduction factor for modulus of elastic and strength at different temperature, \( T_i \).

| Table 2. Parameters of temperatures and reduction parameters |
|---|---|---|---|
| \( i \) | \( T_i [\degree C] \) | \( k_E = k_G \) | \( k_{\text{t,para}} \) |
| 1 | 20 | 1.00 | 1.00 |
| 2 | 100 | 0.425 | 0.65 |
| 3 | 300 | 0.01* | 0.01* |

Note: * 0.01 is suggested to avoid from the convergent problems in the analysis.

### 5.7 Required Parameters

The material properties used in the calculations are as follows (Table 4). They were obtained partially from references and partially derived in the present study. Some of these have been adjusted to comply with earlier test results.

<p>| Table 3. Material properties used in the analysis |
|---|---|---|---|
| Property Number | Notation | Meaning | Unit |
| 1 | ( E_L ) | Modulus's of elasticity | MPa |
| 2 | ( E_R ) | | MPa |
| 3 | ( E_T ) | | MPa |
| 4 | ( G_{LR} ) | Shear modulus' | MPa |
| 5 | ( G_{RT} ) | | MPa |
| 6 | ( G_{RL} ) | | MPa |
| 7 | ( \nu_{LR} ) | Poisson ratios | - |
| 8 | ( \nu_{RL} ) | | - |
| 9 | ( \nu_{RT} ) | | - |
| 10 | ( \nu_{TR} ) | | - |</p>
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<th>Meaning</th>
<th>Unit</th>
</tr>
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<td>$v_{TL}$</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>$v_{LT}$</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>$\sigma_{LO,T}$</td>
<td>Tension strength in L direction</td>
<td>MPa</td>
</tr>
<tr>
<td>14</td>
<td>$\sigma_{LO,C}$</td>
<td>Compression strength in L direction</td>
<td>MPa</td>
</tr>
<tr>
<td>15</td>
<td>$\sigma_{RO,T}$</td>
<td>Tension strength in R direction</td>
<td>MPa</td>
</tr>
<tr>
<td>16</td>
<td>$\sigma_{RO,C}$</td>
<td>Compression strength in R direction</td>
<td>MPa</td>
</tr>
<tr>
<td>17</td>
<td>$\sigma_{TO,T}$</td>
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<td>MPa</td>
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<tr>
<td>18</td>
<td>$\sigma_{TO,C}$</td>
<td>Compression strength in T direction</td>
<td>MPa</td>
</tr>
<tr>
<td>19</td>
<td>$\sigma_{LRo}$</td>
<td>Shear strengths</td>
<td>MPa</td>
</tr>
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<td>20</td>
<td>$\sigma_{RTo}$</td>
<td>Hardening parameters</td>
<td>MPa</td>
</tr>
<tr>
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<td>$\sigma_{TLo}$</td>
<td></td>
<td>MPa</td>
</tr>
<tr>
<td>22</td>
<td>$N_L$</td>
<td>Hardening parameters</td>
<td>-</td>
</tr>
<tr>
<td>23</td>
<td>$C_{\sigma,L}$</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>24</td>
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<td>Failure strain</td>
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Table 3. Material properties used in the analysis – con’t

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<th>Unit</th>
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<td>27</td>
<td>$\epsilon_{Ro}$</td>
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<td>28</td>
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<td>Hardening parameters</td>
<td>-</td>
</tr>
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<td>29</td>
<td>$N_T$</td>
<td>Hardening parameters</td>
<td>-</td>
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<td>$C_{\alpha,T}$</td>
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<td>33</td>
<td>$d_{maxL}$</td>
<td>Maximum damage ratios</td>
<td>-</td>
</tr>
<tr>
<td>34</td>
<td>$d_{maxR}$</td>
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</tr>
<tr>
<td>35</td>
<td>$d_{maxT}$</td>
<td>Maximum damage ratios</td>
<td>-</td>
</tr>
</tbody>
</table>
### Property Number | Notation | Meaning | Unit
---|---|---|---
36 | $d_{max}$ |  |  
37, 39, 41 | $T_1, T_2, T_3$ for $k_{Ei}$ |  | °C
38, 40, 42 | $k_{E1}, k_{E2}, k_{E3}$ |  |  
43, 45, 47 | $T_1, T_2, T_3$ for $k_{G1}$ |  | °C
44, 46, 48 | $k_{G1}, k_{G2}, k_{G3}$ |  |  
49, 51, 53 | $T_1, T_2, T_3$ for $k_{t,para,i}$ |  | °C
50, 52, 54 | $k_{t,para,1}, k_{t,para,2}, k_{t,para,3}$ |  |  
55, 57, 59 | $T_1, T_2, T_3$ for $k_{c,para,i}$ |  | °C
56, 58, 60 | $ak_{c,para,1}, k_{c,para,2}, k_{c,para,3}$ | Temperatures and reduction parameters |  
61, 63, 65 | $T_1, T_2, T_3$ for $k_{t,perpl}$ |  | °C
62, 64, 66 | $k_{t,perpl,1}, k_{t,perpl,2}, k_{t,perpl,3}$ |  |  
67, 69, 71 | $T_1, T_2, T_3$ for $k_{c,perpl,i}$ |  | °C
68, 70, 72 | $k_{c,perpl,1}, k_{c,perpl,2}, k_{c,perpl,3}$ |  |  
73, 75, 77 | $T_1, T_2, T_3$ for $k_{s,para,i}$ |  | °C
74, 76, 78 | $k_{s,para,1}, k_{s,para,2}, k_{s,para,3}$ |  |  
79, 81, 83 | $T_1, T_2, T_3$ for $k_{s,perpl,i}$ |  | °C
80, 82, 84 | $k_{s,perpl,1}, k_{s,perpl,2}, k_{s,perpl,3}$ |  |  

### 6 MATERIAL MODEL FOR STEEL

#### 6.1 Stress-strain relationship at ambient temperature

Steel unlike wood is homogenous and possesses the same mechanical properties in all directions (isotropic). Under both pure tensile and compressive stresses, steel exhibits an elasto-plastic behavior which is indicated by three distinct phases (Figure 13). The first stage depicts a linear response of strains in the steel to applied stresses. In the elastic range, withdrawal of the applied stresses returns steel to its original state with no permanent deformation. Beyond the elastic limit steel yields and the plastic region is initiated. When the applied load is removed after the yield point, complete recovery to its original state is impossible and a permanent deformation results with is known as plastic deformation. The second stage lies within the plastic region where strains continually increase with negligible increase in the stresses forming yield plateau. After the yield plateau, stresses increases significantly against increasing strains over a range called the strain-hardening range. The stain-hardening of steel enhances its capacity beyond the yield strength until its ultimate strength is reached. The
ultimate strength is the maximum strength of steel which can be utilized after which the steel will eventually fail by fracture.

Figure 13. Typical stress-strain curve of steel

Generally, the stress-strain curve of steel can be simplified to an elastic-plastic one. Single value of elastic modulus, $E$, Poisson ratio, $v$, and yield strength, $f_y$, are enough to complete define the elastic response of the steel to stresses. The typical value of $E$ and $v$ for steel are 200 GPa and 0.3, respectively. A similar stress-strain relationship for carbon steel, Figure 14, by EN 1993-1-2 (2005) and also CSA S16 (CSA 2014) was adopted in this study such that the reduction factors for yield strength, proportional limit and modulus of elasticity by EN 1993-1-2, which the effect of creep is implicitly considered and the material models are applicable for heating between 2 and 50 K/min, can be used. The definitions of effective yield strength, proportional limit and slope of linear elastic range are established on the basic characteristic of the stress-strain model for steel at high temperatures proposed by EN1993-1.2. Figure 14 shows that the first part of the curve is a linear line progressing up to the proportional limit $f_{p,T}$ and the elastic modulus $E_{a,T}$ is equal to the slope of this straight-line segment. The second part of the curve depicts the transition from the elastic to the plastic range. This region is formulated by an elliptical progression up to the effective yield strength, $f_{y,T}$. The third part of the curve is a flat yield plateau up to a limiting strain for yield strength. The last part of the curve is characterized by a linear line decreasing to zero stress at the ultimate strain.
Figure 14. Stress-strain relationship for carbon steel (EN 1993-1-2 2005)

Note: \( f_{y,T} \) is effective yield strength; \( f_{p,T} \) is proportional limit; \( E_{a,T} \) is slope of the linear elastic range; \( \varepsilon_{p,T} \) is strain at the proportional limit; \( \varepsilon_{y,T} \) is yield strain; \( \varepsilon_{t,T} \) is limiting strain for yield strength; \( \varepsilon_{u,T} \) is ultimate strain.

In the elastic region up to the proportional limit, Eq. (48) is valid for steel subject to uniaxial stress.

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{bmatrix} =
\begin{bmatrix}
E & -E/\nu & -E/\nu & 0 & 0 & 0 \\
-E/\nu & E & -E/\nu & 0 & 0 & 0 \\
-E/\nu & -E/\nu & E & 0 & 0 & 0 \\
0 & 0 & 0 & G & 0 & 0 \\
0 & 0 & 0 & 0 & G & 0 \\
0 & 0 & 0 & 0 & 0 & G
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{13} \\
\varepsilon_{23}
\end{bmatrix}
\] (48)

The relationship between the stress and strain of steel beyond the proportional limit can be described as EN1993-1-2 provides detailed mathematical formulae of stress-strain relationships of steel.

\[
\sigma = \begin{cases} 
\varepsilon E_{a,T} & \varepsilon \leq \varepsilon_{p,T} \\
\frac{f_{p,T} - c + \frac{b}{a}\sqrt{a^2 - (\varepsilon_{y,T} - \varepsilon)^2}}{f_{y,T}} & \varepsilon_{p,T} < \varepsilon < \varepsilon_{y,T} \\
\frac{f_{y,T} - (\varepsilon - \varepsilon_{y,T})}{\varepsilon_{u,T} - \varepsilon_{y,T}} & \varepsilon_{y,T} \leq \varepsilon < \varepsilon_{t,T} \\
\frac{f_{y,T} - (\varepsilon - \varepsilon_{y,T})}{\varepsilon_{u,T} - \varepsilon_{y,T}} & \varepsilon_{t,T} \leq \varepsilon < \varepsilon_{p,T} \\
0 & \varepsilon \geq \varepsilon_{u,T}
\end{cases}
\] (49)

where \( a, b \) and \( c \) are the function parameters which can be calculated by

\[
a = \sqrt{(\varepsilon_{y,T} - \varepsilon_{p,T})(\varepsilon_{y,T} - \varepsilon_{p,T} + c/E_{a,T})}
\] (50a)
\[ b = \sqrt{c (\varepsilon_{y,T} - \varepsilon_{p,T}) E_{a,T} + c^2} \]  
\[ c = \frac{(f_{y,T} - f_{p,T})^2}{(\varepsilon_{y,T} - \varepsilon_{p,T}) E_{a,T} - 2(f_{y,T} - f_{p,T}) \varepsilon_{p,T}} \]

where \( \varepsilon_{p,T} = \frac{f_{y,T}}{E_{a,T}}, \varepsilon_{y,T} = 0.02, \varepsilon_{e,T} = 0.15, \varepsilon_{u,T} = 0.20 \) according to EN 1993-1-2.

### 6.2 Temperature Influence

The stress-strain behaviour of carbon steel at high temperatures is essentially different from that at ambient temperature, without a clear yield plateau but strain hardening occurring all the way in the plastic range. British Steel Corporation (now named as Corus) carried out an extensive small-scale tensile test program in 1980s on BS4360: Grades 43A and 50B steels to provide elevated temperature data for structural fire engineering design applications. To represent the behaviour of beams and columns in large scale tests, the heating rates were set at the range 5 to 20°C/min. The test results show that carbon steel begins to lose strength at temperatures above 300°C and reduces in strength at a steady rate up to 800°C. The well-defined yield plateau at 20°C is replaced by a gradual increase of strength with increasing strain (or strain-hardening) at high temperatures. Such characteristics make it very difficult to define the strength of steel at high temperatures which is an important parameter in fire structural design.

Based on the British Steel data, EN1993-1.2 derives the reduction factors for effective yield strength, proportional limit and slope of linear elastic range, which have also been incorporated into Annex K of CSA S16, as given in Table 4. The effective yield strength is related to 2% strain limit. Figure 15 illustrates the variation of the reduction factors with temperature for the yield strength, proportional limit and MOE. The strength reduction factor at 2% strain of BS5950-8 is also plotted for comparison.

![Figure 15. Reduction factors for the stress-strain relationship of carbon steel at elevated temperatures (EN 1993-1-2 2005)](image-url)
Table 4. Reduction factors for stress-strain relationship of carbon steel at elevated temperatures

<table>
<thead>
<tr>
<th>Steel Temperature, T [°C]</th>
<th>$k_{y,T} = f_{y,T}/f_y$</th>
<th>$k_{p,T} = f_{p,T}/f_y$</th>
<th>$k_{E,T} = E_{a,T}/E_a$</th>
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<tbody>
<tr>
<td>20</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>100</td>
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<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>200</td>
<td>1.00</td>
<td>0.807</td>
<td>0.900</td>
</tr>
<tr>
<td>300</td>
<td>1.00</td>
<td>0.613</td>
<td>0.800</td>
</tr>
<tr>
<td>400</td>
<td>1.00</td>
<td>0.420</td>
<td>0.700</td>
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<td>500</td>
<td>0.78</td>
<td>0.360</td>
<td>0.600</td>
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<td>0.47</td>
<td>0.180</td>
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<tr>
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<td>0.075</td>
<td>0.130</td>
</tr>
<tr>
<td>800</td>
<td>0.11</td>
<td>0.050</td>
<td>0.090</td>
</tr>
<tr>
<td>900</td>
<td>0.06</td>
<td>0.0375</td>
<td>0.0675</td>
</tr>
<tr>
<td>1000</td>
<td>0.04</td>
<td>0.025</td>
<td>0.045</td>
</tr>
<tr>
<td>1100</td>
<td>0.02</td>
<td>0.0125</td>
<td>0.0225</td>
</tr>
<tr>
<td>1200</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Note: $k_{y,T}$, $k_{y,T}$, and $k_{E,T}$ are reduction factors for effective yield strength, proportional limit, and the slope of the linear elastic range, respectively. Reduction factors at temperature $T$ relative to the value of $f_y$ or $E_a$ at 20°C. For intermediate values of the steel temperature, linear interpolation may be used.

Basically, the slope of linear elastic range governs the steel stiffness, whereas the effective yield strength governs the strength. Comparing their reduction factors at elevated temperatures, it can be seen that the stiffness of steel reduces earlier and more rapid than the strength. This indicates that the failure mode of steel members may change at elevated temperatures.

### 6.3 Required Parameters

The value of elastic modulus, $E$, Poisson ratio, $\nu$, and yield strength, $f_y$, of the steel are required for the analysis. The typical value of $E$ and $\nu$ for steel are 200 GPa and 0.3, respectively.

### 7 MODEL VERIFICATION

To verify and validate the developed material model, two cases, including a LVL beam and a glulam bolted connection, have been modelled with ABAQUS/CAE. The calculated results have been compared to earlier experimental results found in the literature. All the simulations use ABAQUS 6.14 and are run with ABAQUS/Explicit. The hexagonal 3D elements C3D8T have been used to mesh the wood parts and steel parts. The contacts have been modelled by a hard contact pair with a penalty method in the tangential direction using a friction factor of 0.4. Note
the purpose of these comparisons is to validate the developed material model and methods used. The refinement of this model and its parameters will be a future task.

7.1 LVL Beam

Ten separated load-bearing structural composite lumber (SCL) beams were tested under the time-temperature curve of ASTM E119 (ASTM 2016) (WFCi 2014). Single-beam was placed in the center of the horizontal furnace spanning 4.3 m (14 ft) with insulating lids, therefore only the sides and bottom of the beam were exposed to the furnace heat (Figure 16). Each beam was loaded with hydraulic rams to simulate loading of the member. Laminated veneer lumber (LVL), laminated strand lumber (LSL), and parallel strand lumber (PSL) with different widths and grades were tested under various load levels. The influence of gypsum board was investigated by adding gypsum boards to four of the beams (Dagenais 2014).

Figure 16. Schematic of beam showing (a) top view, (b) north-south side view, and (c) east-west side view (WFCi 2014)
Specimen #7 (3100Fb-2.0E Versa-Lam LVL), with a cross-section of 89 x 241 mm without gypsum board and loaded with two point-loads with a total of 16.8 kN (50% of allowable stress ratio), was selected as one of the verification cases in this study. A quarter FE model of this specimen, Figure 17, was developed using ABAQUS/CAE.

Because the mechanical properties of the specimens have not been measured in the test, the mean values of the mechanical properties were obtained by converting the ASD values in APA report (APA 2017) in accordance with ASTM D245, D2915 and D5456, see Appendix II. Table 5 lists the mechanical properties of LVL which were used in the FE model. According to Dagenais (2014), the charring rate of SCL (including PSL, LVL and LSL) is slightly different from solid lumber. In this study, it was assumed that LVL has the same charring rate as solid lumber. The thermal properties, the conductivity and specific heat, and the reducing factors for thermal and mechanical properties of softwood in EN 1995-1-2 were also used for LVL in this study, see Table 5 and Appendix Al.1. The density of LVL was taken as 590 kg/m³ (MC=7%) (APA 2017). The convective heat transfer coefficient, view factor, wood surface emissivity and fire emissivity used in the analysis were also given in Appendix Al.1.
Table 5. Material properties used in the validation

<table>
<thead>
<tr>
<th>Property Number</th>
<th>Notation</th>
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<th>LVL 3100Fb-2.0E</th>
<th>Unit</th>
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<td>13800</td>
<td>MPa</td>
</tr>
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</tr>
<tr>
<td>3</td>
<td>$E_T$</td>
<td>700.4</td>
<td>430</td>
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</tr>
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<td>4</td>
<td>$G_{LR}$</td>
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<td>7</td>
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</tr>
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<td>$v_{RT}$</td>
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<td>$v_{TR}$</td>
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<tr>
<td>11</td>
<td>$v_{TL}$</td>
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<tr>
<td>12</td>
<td>$v_{LT}$</td>
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<td>$C_{a,L}$</td>
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</tr>
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<td>24</td>
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<td>25</td>
<td>$N_R$</td>
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<td>0.4</td>
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<td>$C_{a,R}$</td>
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Table 5. Material properties used in the validation – con’t

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<th>Glulam 20f-EX SP</th>
<th>LVL 3100Fb-2.0E</th>
<th>Unit</th>
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</table>
A static stress/displacement analysis was conducted with the VUMAT subroutine. When a total load of 16.8 kN was applied, the deformation at the centre was 15.1 mm in the FE Model. Compared to the test result of 14.5 mm, it shows the model can reasonably well predict the deformation of the LVL beam under point-loads.

A transient heat-transfer analysis was carried out. The temperature distribution in the cross-section at the centre of the specimen is shown in Figure 18. Figure 19 shows the cross-section views of residual LVL at the middle span from ABAQUS/Standard and the cuts of the residual test specimen. The gray part in the contour presents the charred wood when the temperature is over 300 ºC. The charring rates for the LVL model were 0.79 mm/min (27.8 mm / 35 min) on side and 0.94 mm/min (33.0 mm/ 35 min) at the bottom, respectively. The higher charring rate at the bottom is due to the fact that the bottom material was not only affected by the bottom temperature but also the both sides as well (corner rounding effect). According to Annex B of CSA O86-14, the resulting effective char depth of an LVL beam exposed to a standard fire for 35 min would be taken as 31.5 mm, which would relate to an effective charring rate of 0.90 mm/min. It can be seen that the model gives a fairly good prediction of the residual wood dimensions.

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**Figure 18.** Temperature in the cross-section of LVL after exposing to fire for 35min
A coupled thermal-stress analysis was performed with the developed user subroutine, VUMAT. The deformation of specimen #7 increased with time and increasing temperature, as shown in Figure 20. The charring of the LVL specimen at various times is shown in Figure 21 where elements were charred first then removed once strain met the criterion. The FE model failed at 28.5 mins (Figure 21e), based on the mean properties obtained by converting the ASD values in APA report (APA 2017). The prediction would be expected to be closer to the actual test result if the actual material properties of the specimen were known. With the computed result being within a reasonable variation from the test result of 33.7 mins, this developed FE model and the user-subroutine, VUMAT, could be considered to be capable of simulating the mechanical behaviour of LVL under load or/and fire.
Figure 20. Deflection vs time

(a) Time=0.0 min

(b) Time=2.8 min

(c) Time=17.5 min
7.2 Glulam Connection

In total, 16 bolted wood-steel-wood (WSW) connections and 6 bolted steel-wood-steel (SWS) connections were tested for fire resistance, when exposed to the standard fire CAN/ULC-S101 (Peng et al. 2012a). Figures 22 to 23 show the test setup and a typical WSW specimen before and after the test. All the specimens were subjected to a constant tensile load parallel to grain during the tests. The effects of load level, wood thickness, fastener diameter, number of fasteners, edge distance, and protection were investigated in the tests.
Figure 22. Test setup: (a) Furnace appearance and (b) Front cross-section view (Peng et al. 2012a)
WSW specimen #2.1, consisting of 130 x 190 mm 20f-EX S-P Glulam members, 12.7 mm (1/2") diameter ASTM A307 bolts, and a 9.5 mm (3/8") thick steel plate (grade 300W), was selected as a verification case in this study. A constant tensile load of 11.5 kN, which is equal to 10% of the ultimate load-carrying capacity at ambient conditions (Mohammad and Quenneyville 2001), was applied to the specimen during the fire-resistance test (Peng et al. 2012a). A one-eighth FE model of this specimen, as shown in Figure 24, was developed using ABAQUS.

Figure 23. A typical WSW specimen before (a) and after (b) test (Peng et al. 2012a)
Because the mechanical properties of the specimens have not been measured in the test, the mean values of the mechanical properties were obtained by converting the specified design...
values in CSA O86 (2016) in accordance with the standard procedure (Code committee of CSA O86 2001), ASTM D2915, see Appendix II. The mechanical properties of glulam used in the FE model are given in Table 3. The thermal properties, the conductivity and specific heat, and the reducing factors for thermal and mechanical properties of softwood in EN 1995-1-2 were used for Glulam in this study, see Table 3 and Appendix Al.1. The density of Glulam was taken as 412 kg/m³ (MC=12%) (Peng et al. 2012a). An elasticity of modulus of 2.0x10⁵ MPa and a constant density of 7850 kg/m³ were used for bolts and steel plate. According to CSA O86, specified yield strength of 310 MPa and 450 MPa were used for bolts and steel plate, respectively. The thermal properties, the density, conductivity and specific heat, and the reducing factors for the thermal and mechanical properties of carbon steel in EN 1993-1-2 were used for bolts and steel plate in this study see Table 4 and Appendix Al.2. The convective heat transfer coefficient, view factor, surface emissivity and fire emissivity of wood and steel used in the analysis were also given in Appendix Al.1 and Al.2.

A static stress/displacement analysis was conducted with the VUMAT subroutine. The FE model for the glulam bolt connection yielded at a total tensile load of 89.4 kN. The maximum load of the FE model was 111.3 kN, which is within 5% of the test results (115 kN) by Mohammad and Quenneville (2001). The failure modes obtained from FE model were yield of Glulam and bolts (Figure 25). It shows that the model can predict reasonably well the maximum strength and failure modes of the Glulam bolted connections under ambient conditions.

![Figure 25. Yield modes for specimen WSW #2.1: (a) Glulam; and (b) Bolts](image)
A heat-transfer analysis was carried out. The temperature distribution in the glulam bolted connection is shown in Figure 26.

Figure 26. Temperature in the glulam bolted connection after exposing to fire for 28min: (a) Half model; (b) cross-section with bolts in X-Y plane; (c) cross-section between two bolts in X-Y plane; and (d) cross-section with bolts in Y-Z plane
Figure 27 shows the cross-section views of residual Glulam with/without bolt holes in the X-Y plane from ABAQUS/Standard and the cuts of the residual test specimen. The gray part in the contour presents the charred wood when the temperature is over 300°C. It can be seen that the heat transfer model gives a fairly good prediction of the residual wood dimensions when compared to the test results.
A coupled thermal-stress analysis was performed using the developed user subroutine, VUMAT. The deformation of specimen WSW #2.1 increased with time and increasing temperature, as shown in Figure 28. It can be found that the deflection started to increase after 10 min in the test. This was induced by the wood crushing in the bolt holes due to load and fire. The charring of the Glulam bolted connection at typical time is shown in Figures 29 to 31. The failure model of Glulam member and the deformation of bolts are shown in Figures 32 to 33. The FE model failed at 23.5 min (Figure 31). It is 4.5 min earlier compared to the test result (28.0 min). Therefore, this developed FE model and the user-subroutine, VUMAT, is capable to reasonably simulate the mechanical behaviour of Glulam bolted connection under load or/and fire.
Figure 28. Deflection vs time

Figure 29. Temperature of glulam at ambient temperature (20 °C)
Figure 30. Temperature and charring of glulam under 10% ultimate load and CAN/ULC-S101 fire
Figure 31. Temperature, charring and failure of glulam under 10% ultimate load and CAN/ULC-S101 fire (Time=23.5 min)

Figure 32. Severe hole-elongation: (a) Test; and (b) FEA

Figure 33. Deformation of bolts: (a) FEA; and (b) Test
8 SUMMARY AND CONCLUSIONS

Determination of structural performance of timber connections is a crucial factor in evaluating the overall behaviour of wood structures under force and/or fire. To overcome the lack of suitable models for timber connections, which was found as an impediment to the design and construction of mass timber buildings, a sophisticated constitutive model for wood-based composite material under stress and temperature was developed. This constitutive model was programmed into a user-subroutine, VUMAT, and added to general-purpose finite element software, ABAQUS. The following assumptions are implied in this model:

a) the elastic properties were simplified in the form of orthotropic elasticity;
b) extended Yamada-Sun strength criteria which are able to distinguish the failure/yield in three (longitudinal, radial and tangential) directions were adopted to judge if the material fail/yield in any direction;
c) A strain-based damage evolution was used to describe the post-peak softening of brittle failure due to tension and/or shear;
d) A plastic-flow and a hardening law were derived based on extended Yamada-Sun strength criteria to depict the plastic strain and stress for the yielding due to compression.

The developed model was validated by modelling the structural performance of a laminated veneer lumber (LVL) beam and a glulam bolted connection under force and fire. Compared with the test results, it shows that the developed model was capable of simulating the mechanical behaviour of LVL beam and glulam connection under load or/and fire in a reasonable manner.

With this model, it will allow structural designers to obtain the load-displacement curve of timber connections under force, fire or combination of the two. With this, key design parameters such as capacity, stiffness, displacement and ductility, which are required for seismic or fire design, can be obtained.

To use the model, the thermos-physical and mechanical properties of members and fasteners as well as the impact of temperature on them are required. This information can be found from available codes, research reports and handbooks.

It is recommended that further verification and calibration of the model be conducted on various types of wood products, such as CLT, glulam, SCL and NLT, and fasteners, e.g. screw and rivet. Moreover, a database of the thermal and structural properties of the wood members and fasteners that are commonly used in timber constructions need to be developed to support and facilitate the application of the model.

9 BENEFIT TO MEMBERS, INDUSTRY AND DESIGN COMMUNITY

So far, the constitutive models for depicting the stress-strain relationship of wood-based materials are limited, and cannot properly deal with combined stress and temperature effects on a timber connection. In this project, a sophisticated constitutive model for wood-based composite material under an arbitrary but complex stress (force) and temperature (fire) was
developed. The constitutive model has been programed into a user-subroutine which can be used in most commercially available FE software. With this model, structural designers are able to derive essential and important design parameters for the timber connections, thus developing and designing accurate and reliable structural systems that will provide the required level of structural and fire performance stipulated in building codes.

10 LIST OF SYMBOLS

Symbols

- $a, b, c$: Function parameters for the stress-strain relationship of steel
- $c_d$: Dilatational wave speed
- $c_s$: Specific heat [J/(kg·K)]
- $C_{a,l}, C_{a,L}, C_{a,R}, C_{a,T}$: Hardening-rate parameters (general, longitudinal, radial and tangential)
- $C_{b,l}, C_{b,R}, C_{b,T}$: Second hardening-rate parameters (general, longitudinal, radial and tangential)
- $D_{ijkl}$: Stiffness parameter [MPa]
- $d$: Scalar damage parameter
- $d_i, d_L, d_R, d_T$: Scalar damage parameters (general, longitudinal, radial and tangential)
- $d_{maxL}, d_{maxR}, d_{maxT}$: Maximum damage parameters (longitudinal, radial and tangential)
- $d_{maxLc}$: Maximum damage parameters for compression parallel to grain
- $E$: Modulus of elasticity of steel [MPa]
- $E_L, E_R, E_T$: Modulus of elasticity of wood [MPa]
- $E_{a,T}$: Slope of the linear elastic range of the stress-strain curve for steel [MPa]
- $f_{l,i}, f_{R,i}, f_{T,i}$: Initial yield surface functions (longitudinal, radial and tangential)
- $f_l, f_R, f_T$: Ultimate yield surface functions (longitudinal, radial and tangential)
- $f_{R,f}, f_{T,f}$: Final ultimate yield surface functions (radial and tangential)
- $f_i^*$: Trail elastic yield surface function in the $i$ direction
- $f_{p,T}, f_{y,T}$: Proportional limit and effective yield strength for steel [MPa]
- $f_y, f_u$: Yield and ultimate strengths for steel [MPa]
- $f^n$: Yield surface function at the time increment $n$
- $g$: Plastic potential function
\(G\)  
Shear modulus of steel [MPa]

\(G_{LR}, G_{RT}, G_{TL}\)  
Shear modulus of wood [MPa]

\(G_{a,L}, G_{a,R}, G_{a,T}\)  
Hardening model translational limit functions (longitudinal, radial and tangential)

\(G_{\beta,R}, G_{\beta,T}\)  
Second hardening model translational limit functions (radial and tangential)

\(k\)  
Thermal conductivity [W/(m·K)]

\(k_E, k_G\)  
Temperature reduction factor for modulus of elastic and shear modulus

\(k_{EI}, k_{Gi}\)  
Temperature reduction factor for modulus of elastic and shear modulus at different key temperature point

\(k_{E,t}, k_{E,c}\)  
Temperature reduction factor for tensile and compressive modulus of elastic parallel to grain.

\(k_S, k_{Si}\)  
Temperature reduction factor for strength (general and at different key temperature point)

\(k_{t,para}, k_{c,para}, k_{s,para}\)  
Temperature reduction factor for tensile, compressive and shear strength parallel to grain

\(k_{t,para,i}, k_{c,para,i}, k_{s,para,i}\)  
Temperature reduction factor for tensile, compressive and shear strength parallel to grain at different key temperature point

\(k_{t,perp}, k_{c,perp}, k_{s,perp}\)  
Temperature reduction factor for tensile, compressive and shear strength perpendicular to grain

\(k_{t,perp,i}, k_{c,perp,i}, k_{s,perp,i}\)  
Temperature reduction factor for tensile, compressive and shear strength perpendicular to grain at different key temperature point

\(k_{y,T}, k_{p,T}, k_{E,T}\)  
Temperature reduction factor for effective yield strength, proportional limit, and the slope of the linear elastic range, respectively

\(L_{min}\)  
The smallest element dimension in the mesh

\(m\)  
Mass scaling factor

\(N_L, N_R, N_T\)  
Hardening initial parameters (longitudinal, radial and tangential)

\(n_R, n_T\)  
Second hardening parameters (radial and tangential)

\(\dot{Q}\)  
Internally generated heat by the rate of heat consumption per unit volume due to chemical reaction (pyrolysis of wood) and the heat to evaporate water per unit volume [W/m³]

\(T\)  
Temperature [°C] or [K]
Key temperature point for temperature reduction factor [$^\circ C$]

Heating gas temperature [$^\circ K$]

Receiving surface temperature [$^\circ K$]

Time [min] or [sec]

Time increment

Three directions of Cartesian coordinate system

Angle of the slope of the stress-strain curve of steel

Convection heat transfer coefficient [W/(m²·K)]

Backstress tensor (general, longitudinal, radial and tangential) [MPa]

Maximum backstress tensor (general, longitudinal, radial and tangential) [MPa]

Backstress tensor (general, radial and tangential) of second hardening for wood under compression perpendicular to grain [MPa]

Backstress increment (general, longitudinal, radial and tangential) [MPa]

Backstress increment (general, radial and tangential) of second hardening for compression perpendicular to grain [MPa]

Total, elastic and plastic strain increment

Effective strain increment (general, longitudinal, radial and tangential)

Stress increment and trial stress increment

Plasticity consistency parameters (general, longitudinal, radial and tangential)

Difference between yield surface function at $n$ and $n + 1$ (general, longitudinal, radial and tangential)

Strain tensor

Strain components for an orthotropic material

Initial failure strain under compression parallel to grain

Initial second hardening strain (radial and tangential)

Fire source emissivity

Surface material emissivity

Strain at the proportional limit, yield strain, limiting strain for yield strength, and ultimate strain

The largest eigenvalue in the system of equations of the thermal
solution response

\( \lambda_0, \mu_0 \) Lamé constants

\( \nu, \nu_{ij} \) Poisson's ratio

\( \nu_{LR}, \nu_{LT}, \nu_{RT}, \nu_{RL}, \nu_{TL}, \nu_{TR} \) Poisson's ratios of an orthotropic material (wood notation)

\( \rho \) Density [kg/m\(^3\)]

\( \sigma_{sb} \) Stefan-Boltzmann constant, \( 5.67 \times 10^{-8}, \) [W/(m\(^2\)-K\(^4\)]

\( \sigma \) Stress [MPa]

\( \sigma_{ij}, \sigma_{ij}^*, \sigma_{ij} \) Stress tensors (tensile, elastic and with damage) [MPa]

\( \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31} \) Stress components of an orthotropic material [MPa]

\( \bar{\sigma}_L, \bar{\sigma}_R, \bar{\sigma}_T \) Initial yield strength (longitudinal, radial and tangential) [MPa]

\( \sigma_L^F, \sigma_R^F, \sigma_T^F \) Ultimate yield strength (longitudinal, radial and tangential) [MPa]

\( \sigma_{io}, \sigma_{io,T}, \sigma_{io,C} \) Axial strength, tensile and compressive strength in the \( i \) direction [MPa]

\( \sigma_{L0}, \sigma_{L0,T}, \sigma_{L0,C} \) Longitudinal wood strengths (general, tension, and compression) [MPa]

\( \sigma_{R0}, \sigma_{R0,T}, \sigma_{R0,C} \) Radial wood strengths (general, tension, and compression) [MPa]

\( \sigma_{T0}, \sigma_{T0,T}, \sigma_{T0,C} \) Tangential wood strengths (general, tension, and compression) [MPa]

\( \sigma_{ij0} \) Shear strength in the \( i-j \) plane [MPa]

\( \sigma_{LR0}, \sigma_{RT0}, \sigma_{TL} \) Shear strengths [MPa]

\( \tau_{ij}, \tau_L, \tau_R, \tau_T \) Instantaneous strain energy type term for damage accumulation (general, longitudinal, radial and tangential) [MPa\(^{1/2}\)]

\( \tau_{L0}, \tau_{L0}, \tau_{R0}, \tau_{T0} \) Initial strain energy type term for damage accumulation (general, longitudinal, radial and tangential) [MPa\(^{1/2}\)]

\( \Upsilon \) Factor for stiffness parameters

\( \gamma_{LR}, \gamma_{LR}, \gamma_{LR} \) Shear strain

\( \phi \) View factor

\( \omega \) Moisture content

\( \omega_{max} \) The highest frequency in the system of equations of the mechanical solution response
Subscripts

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<tr>
<td>⊥</td>
<td>Perpendicular</td>
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11 REFERENCES


KERTO. (2017). “Mechanical Properties of KERTO LVL.” KERTO.


APPENDIX I THERMAL PROPERTIES

AI.1 Wood

Two methods can be used to determine the thermal properties of material:

(1) The more common approach is to use the given material properties in a “k-p-c model”, with an implicit consideration of a moisture content of 12% in the density function and as heat of vaporisation in the specific heat function.

(2) Alternatively the moisture content can be considered explicitly as a latent heat or enthalpy for a user-specified moisture content. The latent heat model allows for the removal of this peak from the specific heat curve mentioned above, as it specifically accounts for the extra energy of this phase change over the specified temperature range.

According to EN1995-1-2, values of thermal conductivity, specific heat and the ratio of density to dry density of softwood may be taken as given in Table A1 for standard fire exposure.

Table A1. Wood thermal properties, as presented in EN 1995-1-1

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>Thermal Conductivity [W/(m·K)]</th>
<th>Specific Heat [kJ/(kg·K)]</th>
<th>Ratio of Density to Dry [Density]^a</th>
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<tr>
<td>100^b</td>
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<td>13.60^b</td>
<td>1+ω</td>
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Note: (a) ω is the moisture content; (b) Data is added or modified on purpose to avoid convergence problem during the modelling; (c) Data can be derived using linear interpolation for empty cells.
The variation of the thermal conductivity, specific heat and density ratio with temperature is illustrated in Figure A1 to A3.

![Figure A1. Temperature-thermal conductivity relationship for wood and the char layer](image1)

![Figure A2. Temperature-specific heat relationship for wood and charcoal](image2)
Eurocode 1: Part 1-2 and Eurocode 5: Part 1-2 suggest that, in the case of a standard time-temperature curve exposure, the convective heat transfer coefficient, view factor, wood surface emissivity and fire emissivity be taken as 25 W/(m²·K), 1.0, 0.8, and 1.0, respectively, which have been used in this research.

**A1.2 Steel**


The thermal conductivity of steel, \(k\), [W/(m·K)] should be determined from Equation (A1).

\[
k = \begin{cases} 
54 - 3.33 \times 10^{-2} T & 20^\circ C \leq T \leq 800^\circ C \\
27.3 & 800^\circ C \leq T \leq 1200^\circ C 
\end{cases}
\]  

where \(T\) is the steel temperature \(\left[^\circ C\right]\). The variation of the thermal conductivity with temperature is illustrated in Figure A4.
The specific heat of steel, $c_s$, [J/(kg·K)] should be determined from Equation (A2).

$$c_s = \begin{cases} 
425 + 7.73 \times 10^{-1} T - 1.69 \times 10^{-3} T^2 + 2.22 \times 10^{-6} T^3 & 20^\circ C \leq T \leq 600^\circ C \\
666 + \frac{13002}{738-T} & 600^\circ C \leq T \leq 735^\circ C \\
545 + \frac{17820}{T-731} & 735^\circ C \leq T \leq 900^\circ C \\
650 & 900^\circ C \leq T \leq 1200^\circ C 
\end{cases}$$  \hspace{1cm} (A2)

The variation of the specific heat with temperature is illustrated in Figure A5.
The unit mass of steel, $\rho$, is considered to be independent of the steel temperature. It can be taken as $7850 \text{ kg/m}^3$.

Eurocode 1: Part 1-2 and Eurocode 3: Part 1-2 suggest that, in the case of a standard time-temperature curve exposure, the convective heat transfer coefficient, view factor, steel surface emissivity and fire emissivity be taken as $25 \text{ W/(m}^2\text{K)}$, 1.0, 0.8, and 1.0, respectively, which have been used in this research.
APPENDIX II MECHANICAL PROPERTIES

Two types of mechanical properties can be used as input for the timber connection modelling depending on the analysis purpose. Test results (mean values) are often used to investigate the relatively-real response of timber connections under forces and fire. With respect to structural and fire design, design values, also call characteristic values or lower bound properties, may be used for the modelling. For limit states design (LSD), the specified strength design values of wood-based products are given in CSA O86 Engineering Design in Wood (CSA 2016). When analyzing following the allowable stress design methodology (ASD), the allowable strength values may be found in the National Design Specification for Wood Construction (AWC 2016). Whether the analysis is made per LSD or ASD, proper strength and modification factors are to be applied as per the applicable wood design standard.

Moreover, some average tested mechanical properties of Canadian and American-grown species of commercial importance are provided by “Strength and Related Properties of Woods Grown in Canada” (Jessome 2000) and “Wood Handbook – Wood as an Engineering Material” (FPL 2000), respectively. The data were derived from tests performed on small clear specimens of lumber, which are free from defects such as knots, cross-grain, decay, checks, shakes, wane, or reaction wood. They could be used as input for the modelling of timber connections under forces and fire. However, the influence of defects and other parameters should be taken into account.

Figure A6 provides schemes for converting allowable design stress and specified strength to mean value. Examples showing the derivation of the mean values of the tensile and compressive strength of LVL and Glulam used in the variation are given below.

![Figure A6](image-url)
Example 1: To determine the mean values of the tensile and compressive strength of LVL based on allowable design stress.

(a) Tensile strength
   a) \( F_a = 14.8 \text{ MPa} \) (3100Fb-2.0E Versa-Lam LVL, APA PR L266)
   b) \( S = 14.8 / (1/1.6) / (1/0.86) = 20.4 \text{ MPa} \) (1.6 and 0.86 are factors for DOL and size effect)
   c) \( X_{\text{sth}} = 20.4 \times 2.1 = 42.8 \text{ MPa} \) (2.1 is adjustment factor)
   d) \( \bar{x} = 42.8 / (1 - 1.645 \times 15\%) = 56.4 \text{ MPa} \) (15\% is assumed COV for LVL products)

(b) Compressive strength
   a) \( F_a = 20.7 \text{ MPa} \) (3100Fb-2.0E Versa-Lam LVL, APA PR L266)
   b) \( S = 20.7 / (1/1.6) / (1/1.0) = 33.1 \text{ MPa} \) (1.6 and 1.0 are factors for DOL and size effect)
   c) \( X_{\text{sth}} = 33.1 \times 1.9 = 62.9 \text{ MPa} \) (1.9 is adjustment factor)
   d) \( \bar{x} = 62.9 / (1 - 1.645 \times 15\%) = 83.5 \text{ MPa} \) (15\% is assumed COV for LVL products)

Example 2: To determine the mean values of the tensile and compressive strength of Glulam based on specified strength.

(a) Tensile strength
   a) \( R_s = 17.0 \text{ MPa} \) (20f-EX S-P Glulam, CSA O86)
   b) \( R_n = 17.0 / 0.8 = 21.3 \text{ MPa} \) (0.8 is a factor demonstrating load duration equivalence)
   c) \( X_{\text{sth}} = 21.3 / 1.05 = 20.2 \text{ MPa} \) (1.05 is a reliability normalization factor)
   d) \( \bar{x} = 20.2 / (1 - 1.645 \times 18\%) = 28.8 \text{ MPa} \) (18\% is assumed COV for Glulam)

(b) Compressive strength
   a) \( R_s = 25.2 \text{ MPa} \) (20f-EX S-P Glulam, CSA O86)
   b) \( R_n = 25.2 / 0.8 = 31.5 \text{ MPa} \) (0.8 is a factor demonstrating load duration equivalence)
   c) \( X_{\text{sth}} = 31.5 / 0.98 = 32.2 \text{ MPa} \) (0.98 is a reliability normalization factor)
   d) \( \bar{x} = 32.2 / (1 - 1.645 \times 18\%) = 45.8 \text{ MPa} \) (18\% is assumed COV for Glulam)
APPENDIX III A BRIEF LITERATURE REVIEW OF INVESTIGATION ON STRUCTURAL PERFORMANCE OF TIMBER CONNECTIONS EXPOSED TO FIRE

In total, 43 publications including papers and theses, which investigated the structural performance of timber connections exposed to fire, have been reviewed and listed in Table A2 with some important information, e.g. connection type, type of wood component and fastener type. Here is a brief summary:

(a) Most specimens were common connections used for connecting two wood components, some beam-to-column and primary-beam-to-secondary-beam connections were also tested as well;
(b) Wood members were typically made from LVL and Glulam, while sawn lumber was used in some cases, CLT was only used as a wall for primary-beam-to-secondary-beam connection test; and
(c) Bolts and dowels were used a lot in the tested specimens, and screws, nails and self-tapping screw were used in some cases only.

Table A2. A summary of research on structural performance of timber connections exposed to fire

<table>
<thead>
<tr>
<th>No</th>
<th>Year</th>
<th>Country</th>
<th>Title</th>
<th>Authors</th>
<th>Connection Type</th>
<th>Wood Species</th>
<th>Fastener Type</th>
<th>Research Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1991</td>
<td>New Zealand</td>
<td>Fire Performance of Gusset Connections in Glue-laminated Timber</td>
<td>Buchanon &amp; King</td>
<td>SWS</td>
<td>Glulam</td>
<td>Gusset Plate</td>
<td>Test</td>
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<tr>
<td>2</td>
<td>1996</td>
<td>Sweden</td>
<td>Load-bearing capacity of nailed joints exposed to fire</td>
<td>Noren</td>
<td>WWW</td>
<td>Sawn Lumber</td>
<td>Nails</td>
<td>Test</td>
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<tr>
<td>3</td>
<td>1999</td>
<td>France</td>
<td>Vérification expérimentale de la résistance au feu des assemblages d'éléments en bois (Experimental verification of the fire resistance of joints of wooden elements)</td>
<td>Dhima</td>
<td>WWW &amp; WSW</td>
<td>Glulam</td>
<td>Bolts &amp; Dowels</td>
<td>Test</td>
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<td>4</td>
<td>2001</td>
<td>Austria</td>
<td>Fire behaviour of steel and fasteners in wood composites</td>
<td>Fornath er et al.</td>
<td>WSW</td>
<td>Wood Composite</td>
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<td>Test</td>
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<td>2003</td>
<td>France</td>
<td>Comportement au feu d’assemblages bois (Fire behavior of timber joints)</td>
<td>Ayme</td>
<td>WSW &amp; WSW</td>
<td>Glulam</td>
<td>Dowels &amp; Bolts</td>
<td>Test</td>
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<tr>
<td>6</td>
<td>2006</td>
<td>France</td>
<td>Thermo-mechanical analysis of the timber connection under fire using a finite element model</td>
<td>Laplance he et al.</td>
<td>WSW &amp; WWW</td>
<td>Glulam</td>
<td>Dowels &amp; Bolts (Combined)</td>
<td>Modelling</td>
</tr>
<tr>
<td>8</td>
<td>2006</td>
<td>Switzerland</td>
<td>Thermal investigations on multiple shear steel-to-timber connections</td>
<td>Erchinger et al.</td>
<td>WSSWSW</td>
<td></td>
<td>Dowels</td>
<td>Modelling</td>
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<td>Fire resistance of timber connections</td>
<td>Austruy</td>
<td>SWS, WSW &amp; WWW</td>
<td>LVL</td>
<td>Bolts</td>
<td>Test &amp; Modelling</td>
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<td>Chuo</td>
<td>WWW, WSW &amp; SWS</td>
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<td>Test</td>
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<td>On the design of timber bolted connections subjected to fire</td>
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<td>Fire performance of bolted connections in laminated veneer lumber (LVL)</td>
<td>Lau et al.</td>
<td>WWW, WSW &amp; SWS</td>
<td>LVL</td>
<td>Bolts</td>
<td>Test</td>
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Table A2. A summary of research on structural performance of timber connections exposed to fire – con’t

<table>
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<th>Fastener Type</th>
<th>Research Type</th>
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<td>2011</td>
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<td>Numerical investigations on the thermo-mechanical behavior of steel-to-timber joints exposed to fire</td>
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<td>Glulam</td>
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<td>Modelling</td>
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<td>Behavior of dowelled and bolted steel-to-timber connections exposed to fire</td>
<td>Audebe rt et al.</td>
<td>WSW</td>
<td>Glulam</td>
<td>Dowels &amp; Bolts (Combined)</td>
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<td>26</td>
<td>2012</td>
<td>France</td>
<td>Experimental and Numerical Analysis of the Thermo-Mechanical Behavior of Steel-to-Timber Connections in Tension Perpendicular to the Grain</td>
<td>Audebe rt et al.</td>
<td>WSW</td>
<td>Glulam</td>
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<td>Test &amp; Modelling</td>
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<td>France</td>
<td>Experimental and Numerical Analysis of the Thermo-Mechanical Behaviour of Steel-to-Timber Connections in Bending</td>
<td>Dhima et al.</td>
<td>WSW &amp; WWW</td>
<td>Glulam</td>
<td>Dowels &amp; Bolts (Combined)</td>
<td>Test &amp; Modelling</td>
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<tr>
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<td>2013</td>
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<td>Thermo-mechanical behaviour of timber-to-timber connections exposed to fire</td>
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<td>Glulam</td>
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<td>Frang et al.</td>
<td>WSW &amp; SWS</td>
<td>LVL</td>
<td>Dowels</td>
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<td>Glulam &amp; Steel</td>
<td>Bolts</td>
<td>Test</td>
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<tr>
<td>32</td>
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<td>Experimental and numerical analysis of timber connections in tension perpendicular to grain in fire</td>
<td>Audebe rt et al.</td>
<td>WSW</td>
<td>Glulam</td>
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<td>No.</td>
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<td>Analysis of the behavior of multiple dowel timber connections in fire</td>
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<td>Fire Behaviour of Blind Dovetail Timber Connections</td>
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<td>Analysis of the Thermo-Mechanical Behaviour of Steel-to-Timber Connections in Bending</td>
<td>Dhima et al.</td>
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<td>Glulam</td>
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<td>Glulam</td>
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<td>Full Scale Tests on the Performance of Hybrid Timber Connections In Real Fires</td>
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<td>Glulam</td>
<td>Bolts</td>
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<td>Canada</td>
<td>Performance of Timber-to-Steel Bolted Connections Exposed to Fire</td>
<td>Alam et al.</td>
<td>WSW &amp; SWS</td>
<td>Glulam</td>
<td>Bolts</td>
<td>Test &amp; Modelling</td>
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<td>42</td>
<td>2016</td>
<td>China</td>
<td>Fire Behaviour of Dowel-Type Timber Connection with Slotted in Steel Plates Under Bending and Shear</td>
<td>Zhang et al.</td>
<td>WSW</td>
<td>Glulam</td>
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APPENDIX IV USER’S MANUAL

AIV.1 Requirement
The finite element analysis method presented in this report has been developed with general-purpose finite element software, ABAQUS, and its use requires

- An effective computer
- ABAQUS and an ABAQUS license
- Fortran and a Fortran license compatible with the ABAQUS version
- The implementation scheme of the stress-strain relation for wood into the FE-program (VUMAT subroutine for ABAQUS)

The last component of the list above has been developed at FPIInnovations.

AIV.2 Modelling of Timber Connections with ABAQUS
At present, there are no ready-made wood models in ABAQUS for structural analysis, however there is a way to create own material models. ABAQUS uses several subroutines, programmed in FORTRAN language, to permit the users to define their own material model. In our case, it is necessary to use the VUMAT subroutine to implement the constitutive model of wood under force and fire. The following scheme (Figure A7) describes the modelling and calculating processes used for modelling wood in the present study.

![Figure A7. Scheme of timber connection modelling process](image-url)
AIV.2.1 Modelling with ABAQUS CAE

As shown in the previous scheme (Figure A7), the user can define the geometry, the boundary conditions, the material properties, the (mechanical and fire) loads and the mesh with ABAQUS/CAE. Applying the following steps, it is possible to create a model of timber connection:

- **Enter the part field,**
  - create the part(s), note that to create a glulam beam, it is recommended to first create the whole beam and then to divide it in several lamellae. This will avoid the discontinuities in results between lamella.

- **Enter the property field,**
  - create the material(s); for the wood, create only the name, the material properties can be given directly in the input file by copy-pasting the material properties from the previous input file,
  - create as many sections as materials in the structure, define which material is used for each section,
  - attribute to each part of the structure the correspondent section,
  - create a local coordinate system for each piece of wood,
  - attribute a material orientation for each piece of wood using the local coordinate system.

- **Enter the assembly field,**
  - instance the part and select all the parts of the structure, cross the option “independent”.

- **Enter the step field,**
  - create as many step as needed for the load case, here select the kind of analyses, select the coupled temperature-displacement analyses, then define the time period of the step, the maximum number of increment, the initial, minimum and maximum increment, the maximum temperature increment, define the kind of load (instantaneous or ramp),
  - create a field output, select the results wanted in the output file and define the frequency of saving.

- **Enter the interaction field,**
  - in case of glued parts, create a constraint, select the “Tie” constraint, and select the surfaces in contact,
  - in case of sliding contact, create an interaction, select “surface to surface contact”, select the finer mesh or the stiffer material as master surface, select the option small sliding and then surface to surface (this option permits to adjust the geometry to avoid the initial overclosure), select the option “adjust only to remove overclosure” or create a set of surfaces to be adjusted (the slave surfaces for example) and use the option “adjust slave nodes in set”, select an interaction property or create one if necessary,
create an interaction property, select “contact”, then, in the mechanical field, define the tangential and normal behaviour, for simple contact definition, it is recommended to use frictionless or penalty method for tangential behaviour, and hard contact for normal behaviour, select “allow separation after contact”.

- Enter the load field,
  - create a field, this permits to define an initial temperature, select the initial step, select the wood parts needed and give the initial temperature magnitude,
  - create load, select the step and the kind of load, select “thermal” and “surface heat flux” to define the surfaces of temperature exchange between wood and air,
  - create a boundary condition, select the step and the kind of boundary condition needed,
  - to define the initial temperature, create a field, select the initial step, select the temperature option and select the whole part which has an initial temperature to be defined, and then give its value,
  - in case of contact analysis, it is strongly recommended to create a smooth amplitude for the loading, it will help the calculation to converge,
  - some techniques can be used to facilitate the convergence of the analysis, see paragraph “Stability of the analysis” below.

- Enter the mesh field,
  - seed the part instance,
  - assign element type, use “coupled temperature displacement” family for transient analyses, it is recommended to use non-reduced quadratic element (C3D8T, C3D20T), they permit to obtain a more accurate moisture calculation,
  - mesh the part instance.

- Enter the job field,
  - create a job,
  - in the job manager, create the input file, note that the input file will be saved in the ABAQUS work folder defined during the installation of the software.

At this stage the input file is created but not usable in this state, a few changes have to be done to link the input file to the subroutines.

**AIV.2.2 Editing the Input File**

Before editing the input file, it is important to understand the file structure. The best way to do that is to understand the main key words used by ABAQUS. Usually, the keywords appear in this order in the input file:

*Node: all the nodes created for the mesh are defined at their initial position,

*Element: the elements are created using the node labels and the element type,
*Nset; *Elset; *Surface: the node sets, element sets and surface sets defined for the different load cases and boundary conditions are created,

*Material: for each material, this key word introduces the material definition. It may be coupled with some other keywords depending on the needs of the calculation,

*Boundary: definition of the boundary conditions,

*Initial conditions: definition of the initial conditions like initial temperature for example,

*Step: this keyword introduces a step definition; there is one step definition for each step, and for each definition the load case and the outputs are defined.

To link the input file and the subroutines, it is necessary to modify the material and the step definitions. In the material definition, the material parameters have to be defined, for this it is necessary to use the following key words: *Conductivity, *Density, *Depvar, *Specific heat, *User material and define the required parameters. If the steps defined in the previous paragraph have been followed, it is only necessary to copy-paste the user material parameters from another input file, and then add *Field key word in the step definition:

*FIELD,NUMBER=1 surface_set_name, 0

In order to define the surface_set_name, find the keyword *INITIAL CONDITIONS and copy-paste the surface set name used for this option.

**AIV.2.3 User Subroutines for ABAQUS**

A subroutine here is an algorithm written by the user in FORTRAN language. It uses ABAQUS state variables to calculate user’s variables and to modify the ABAQUS state variables if necessary. This algorithm runs in parallel to the ABAQUS solver. At each time increment there is an exchange of information between them.

For wood modelling, a VUMAT subroutine is used. It calculates the stress increment for each time increment. According to the ABAQUS documentation (ABAQUS 2016), the VUMAT subroutine

- is used to define the mechanical constitutive behaviour of a material;
- will be called for blocks of material calculation points for which the material is defined in a user subroutine;
- can use and update solution-dependent state variables; and
- can use any field variables that are passed in.

For modelling wood, it is necessary to access and/or to modify specific variables at each time step. More details related to variables to be defined, variables that can be updated and variables passed in for information can be found from ABAQUS User Subroutines Reference Guide (ABAQUS 2016).
AIV.2.4 Defining the Fire
Please refer to “fire model” above.

AIV.2.5 Stability of the analysis
In order to facilitate the stability of the analysis, one is advised to get familiar with specific analyzing technical skills topics, including “conditional stability of the explicit method”, “mass scaling”, “contact” and “viscous stabilization”. All can be found from ABAQUS documentation (ABACUS 2016).

In Abaqus/Explicit the mechanical solution response is obtained using the explicit central-difference integration rule with a lumped mass matrix, and the heat transfer equations are integrated using the explicit forward-difference time integration rule. The explicit procedure integrates through time by using many small time increments. The central-difference and forward-difference operators are conditionally stable, thus the time increments to be as close as possible to the stability limit without exceeding it. The stability limit for both operators (with no damping in the mechanical solution response) is obtained by choosing

\[ \Delta t \leq \min\left(\frac{2}{\omega_{\text{max}}}, \frac{2}{\lambda_{\text{max}}}\right) \]  \hspace{1cm} (A3)

where \( \omega_{\text{max}} \) is the highest frequency in the system of equations of the mechanical solution response and \( \lambda_{\text{max}} \) is the largest eigenvalue in the system of equations of the thermal solution response.

An approximation to the stability limit is often written as the smallest transit time of a dilatational wave across any of the elements in the mesh

\[ \Delta t \approx \frac{l_{\text{min}}}{c_d} \]  \hspace{1cm} (A4)

Similarly, an approximation to stability limit for the forward-difference operator in the thermal solution response is given by

\[ \Delta t \approx \frac{l_{\text{min}}^2}{2\alpha} \]  \hspace{1cm} (A5)

where \( l_{\text{min}} \) is the smallest element dimension in the mesh; \( c_d \) is the dilatational wave speed in terms of Lamé constants, \( \lambda_0 \) and \( \mu_0 \); \( \alpha \) is the thermal diffusivity of the material, is taken as the ratio of the thermal conductivity, \( k \), to the product of the density, \( \rho \), and the specific heat, \( c_s \). The dilatational wave speed can be calculated by

\[ c_d = \sqrt{\frac{\lambda_0 + 2\mu_0}{\rho}} \]  \hspace{1cm} (A6)

In an isotropic, elastic material the effective Lamé constants can be defined in terms of Young’s modulus, \( E \), and Poisson’s ratio, \( \nu \), by

\[ \lambda_0 = \frac{Ev}{(1+\nu)(1-2\nu)} \]  \hspace{1cm} (A7)
\[
\mu_0 = \frac{E}{2(1+v)}
\]

The time increment used in an analysis must be smaller than the stability limits of the central- and forward-difference operators. Failure to use such a time increment will result in an unstable solution.

Mass scaling and sub-cycling are the two methods for reducing the computational cost. Artificially increasing the material density by a factor \( m^2 \), just like decreasing \( T \) to \( T/m \). This concept, called “mass scaling”, reduces the ratio of the event time to the time for wave propagation across an element while leaving the event time fixed, which allows rate-dependent behaviour to be included in the analysis. As another alternative, one can use mixed time integration or “sub-cycling” methods to reduce the computational cost. Reducing the time period artificially will introduce two possible errors. If the simulation speed is increased too much, the increased inertia forces will change the predicted response (in an extreme case the problem will exhibit wave propagation response). The only way to avoid this error is to choose a speed-up that is not too large. The other error is that some aspects of the problem other than inertia forces – for example, material behaviour – may also be rate dependent. In this case that actual time period of the event being modelled cannot be changed. However, the thermal solution response in a fully coupled thermal-stress analysis is not affected by mass scaling.

**AIV.2.6 Running Calculations using Subroutines**

When the input file and the subroutines are created, the calculation is running using an MS-Dos window with MS-Windows System or a command window with a UNIX or LINUX station. It is necessary to save the input file and the Fortran file in the same folder. The command to run the calculation is:

ABAQUS job=input_file_name user=Fortran_file_name

It is possible to add “int” at the end of this command line; it permits to display some information in the command window for each increment.